SOIL MECHANICS

Theoretical: 2hrs/week; Tutorial: 1hrs/week; Laboratory: 2hrs/week.

First semester

No	Title	hr
1	Introduction, Soil formation, Composition and Description of individual Soil Particles.	3
2	Physical Relationships (Phase relationships).	6
3	Nature of Water in Clay, Consistency and Atterberg Limits	6
4	Particle Size Distribution	3
5	Soil Classification (AASHTO and Unified).	3
6	Soil Compaction	3
7	Seepage, One Dimensional Flow, Permeability, Two Dimensional Flow, Flow net	6

Second semester

No	Title	hr
8	Stresses within a Soil Mass, Total and Effective Stresses.	12
9	Compressibility of Soils, Theory of Consolidation, Odometer Test, Consolidation Settlement	12
10	Shear Strength of Soils, Coulomb Shear Strength Equation	6

References

1- Craig R. F, (2004), “Craig's Soil Mechanics”, Seventh Edition, Department of Civil Engineering, University of Dundee UK, Taylor & Francis Group.

2- Braja. M. Das, (2007): “Advanced Soil Mechanics”, Third Edition, Taylor & Francis e-Library.

3- Braja. M. Das, (2006): "Principles of geotechnical Engineering”, Fifth Edition, Nelson, a division of Thomson Canada Limited.

Unit Conversion Table

	To convert from	to		Multiply by
1	in	m		0.0254
2	ft	m		0.3048
3	in^2	mm^2		645.16
4	ft^2	m^2		0.092903
5	in^3	m^3		$16.36*10^{-6}$
6	ft^3	m^3		$28.316*10^{-3}$
7	**Quarter(U.S) liquid**	Liter		0.94635
8	**Gallon (U.S liquid)**	m^3		$3.875*10^{-3}$
9	in^4	cm^4		41.62
10	cm^4	m^4		$1*10^{-8}$
11	ft^4	m^4		$8.36*10^{-3}$
12	gram	Dyne		980.665
13	kg	N		9.8066
14	Ib	Kg		0.45359
15	Kips (1000 Ibs)	kN		4.448
16	Kip/ft	kN/m		14.5939
17	Ib/ft	kg/m		1.488
18	kg/m^2	N/m^2	Pascal	9.8066
19	kg/cm^2	kN/m^2	Kpascal	98.066
20	Kip/ft^2	kN/m^2		47.88
21	$Ib//in^2$(psi)	kN/m^2		6.894
22	Ib.in (torque)	N.m		0.112985
23	Ib.ft	N.m		1.3558
24	Kip.ft	kN.m		1.3558
25	ft.Ib(energy)	Joule		1.3558
26	Cal/g	Joule		4.1868
27	Ib/ft^3	kg/m^3		16.01846
28	Kip/ft^3	kN/m^3		157.0876
29	g/cm^3	Ib/ft^3		62.427
30	g/cm^3	kN/m^3		9.8066

SOIL MECHANICS

Unit One
Basic characteristics of soils

1. THE NATURE OF SOILS:

To the civil engineer, soil is any uncemented or weakly cemented accumulation of mineral particles formed by the weathering of rocks, the void space between the particles containing water and/or air. Weak cementation can be due to carbonates or oxides precipitated between the particles or due to organic matter. If the products of weathering remain at their original location they constitute a residual soil. If the products are transported and deposited in a different location they constitute a transported soil, the agents of transportation being gravity, wind, water and glaciers. During transportation the size and shape of particles can undergo change and the particles can be sorted into size ranges. In British Standards the size ranges detailed in Figure (1) are specified. In Figure (1) the terms 'clay', 'silt', etc. are used to describe only the sizes of particles between specified limits.

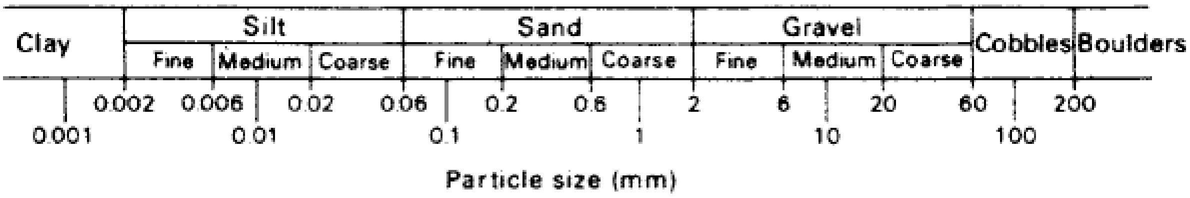


Fig. 1: Particle size ranges.

2. PHASE RELASHIONSHIPS:

Soils can be of either two-phase or three-phase composition. In a completely dry soil there are two phases, namely the solid soil particles and pore air. A fully saturated soil is also two phase, being composed of solid soil particles and pore water. A partially saturated soil is three phase, being composed of solid soil particles, pore water and pore air. The components of a soil can be represented by a phase diagram as shown in Figure 2 (a). The following relationships are defined with reference to Figure 2 (a).

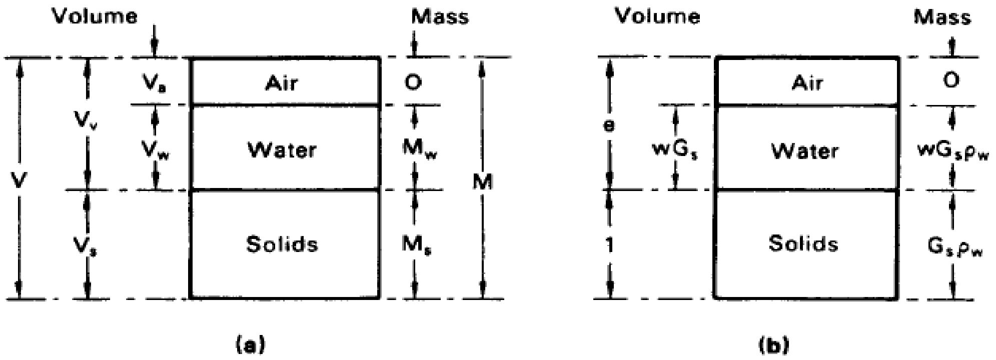


Fig. 2: Phase diagram

The water content (w), or moisture content (m), is the ratio of the mass of water to the mass of solids in the soil, i.e.

$$w = \frac{M_w}{M_s}$$

The degree of saturation (S_r or S) is the ratio of the volume of water to the total volume of void space, The degree of saturation can range between the limits of zero for a completely dry soil and 1 (or 100%) for a fully saturated soil i.e.

$$S_r = \frac{V_w}{V_v}$$

The void ratio (e) is the ratio of the volume of voids to the volume of solids, i.e. $e = \frac{V_v}{V_s}$

The porosity (n) is the ratio of the volume of voids to the total volume of the soil, i.e. $n = \frac{V_v}{V}$

The void ratio and the porosity are inter-related as follows: $e = \frac{n}{1-n}$ $n = \frac{e}{1+e}$

The specific volume (v) is the total volume of soil which contains unit volume of solids, i.e. $v = 1 + e$

The air content or air voids (A) is the ratio of the volume of air to the total volume of the soil, i.e. $A = \frac{V_a}{V}$

The bulk density (ρ) of a soil is the ratio of the total mass to the total volume, i.e. $\rho = \frac{M}{V}$

Convenient units for density are kg/m^3 or Mg/m^3. The density of water (1000 kg/m^3 or Mg/m^3) is denoted by ρ_w.

The specific gravity of the soil particles (G_s) is given by $G_s = \frac{M_s}{V_s \rho_w} = \frac{\rho_s}{\rho_w}$

From the definition of void ratio, if the volume of solids is 1 unit then the volume of voids is e units. The mass of solids is then Gs_w and, from the definition of water content, the mass of water is $wG_s\rho_w$. The volume of water is thus wG_s. These volumes and masses are represented in Figure 2(b). The following relationships can now be obtained.

The degree of saturation can be expressed as:

$$S_r = \frac{wG_s}{e}$$

In the case of a fully saturated soil, S_r =1; hence $e = wG_s$

The air content can be expressed as $A = \frac{e - wG_s}{1+e}$

from $n = \frac{e}{1+e}$ and $S_r = \frac{wG_s}{e}$ $A = n(1 - S_r)$

and

$$\rho = \frac{G_s(1+w)}{1+e}\rho_w \qquad \rho = \frac{G_s + S_r e}{1+e}\rho_w \qquad \gamma = \frac{G_s + S_r e}{1+e}\gamma_w \qquad \gamma = \frac{G_s(1+w)}{1+e}\gamma_w$$

For dry soil S_r = 0 and for fully saturated soil S_r = 1

where γ_w is the unit weight of water. Convenient units are kN/m^3, the unit weight of water being 9.81 kN/m^3 (or 10.0 kN/m^3 in the case of sea water). When a soil in situ is fully saturated the solid soil particles (volume: 1 unit, weight: $G_s\gamma_w$) are subjected to upthrust (γ_w). Hence, the buoyant unit weight (γ') is given by

$$\gamma' = \frac{G_s\gamma_w - \gamma_w}{1+e} = \frac{G_s - 1}{1+e}\gamma_w \qquad \gamma' = \gamma_{sat} - \gamma_w$$

Soil	G_s
Gravel	2.65–2.68
Sand	2.65–2.68
Silt, inorganic	2.62–2.68
Clay, organic	2.58–2.65
Clay, inorganic	2.68–2.75

Derivation is as follows

$$e = \frac{V_v}{V_s} \quad \rightarrow \text{void ratio}$$
$$e = \frac{V_v}{V - V_v}$$
$$e = \frac{V_v}{V - V_v} \cdot \frac{1/V}{1/V}$$
$$e = \frac{V_v/V}{1 - V_v/V} \quad \rightarrow n = V_v / V$$
$$e = \frac{n}{1-n} \quad \textit{(okay!)}$$

$$n = \frac{V_v}{V} \quad \rightarrow \text{porosity}$$
$$n = \frac{V_v}{V_s + V_v}$$
$$n = \frac{V_v}{V_s + V_v} \cdot \frac{1/V_s}{1/V_s}$$
$$n = \frac{V_v/V_s}{1 + V_v/V_s} \quad \rightarrow e = V_v / V_s$$
$$n = \frac{e}{1+e} \quad \textit{(okay!)}$$

Type of soil	Void ratio, e	Natural moisture content in a saturated state (%)	Dry unit weight, γ_d lb/ft^3	Dry unit weight, γ_d kN/m^3
Loose uniform sand	0.8	30	92	14.5
Dense uniform sand	0.45	16	115	18
Loose angular-grained silty sand	0.65	25	102	16
Dense angular-grained silty sand	0.4	15	121	19
Stiff clay	0.6	21	108	17
Soft clay	0.9–1.4	30–50	73–93	11.5–14.5
Loess	0.9	25	86	13.5
Soft organic clay	2.5–3.2	90–120	38–51	6–8
Glacial till	0.3	10	134	21

Example 1:

In its natural condition a soil sample has a mass of 2290 g and a volume of 1.15×10^{-3} m^3. After being completely dried in an oven the mass of the sample is 2035 g. The value of G_s for the soil is 2.68. Determine the bulk density, unit weight, water content, void ratio, porosity, degree of saturation and air content.

$$\text{Bulk density, } \rho = \frac{M}{V} = \frac{2.290}{1.15 \times 10^{-3}} = 1990\,\text{kg/m}^3 \quad (1.99\,\text{Mg/m}^3)$$

$$\text{Unit weight, } \gamma = \frac{Mg}{V} = 1990 \times 9.8 = 19\,500\,\text{N/m}^3$$
$$= 19.5\,\text{kN/m}^3$$

$$\text{Water content, } w = \frac{M_w}{M_s} = \frac{2290 - 2035}{2035} = 0.125 \text{ or } 12.5\%$$

From Equation 1.17,

$$\text{Void ratio, } e = G_s(1 + w)\frac{\rho_w}{\rho} - 1$$

$$= \left(2.68 \times 1.125 \times \frac{1000}{1990}\right) - 1$$

$$= 1.52 - 1$$

$$= 0.52$$

$$\text{Porosity, } n = \frac{e}{1 + e} = \frac{0.52}{1.52} = 0.34 \text{ or } 34\%$$

$$\text{Degree of saturation, } S_r = \frac{wG_s}{e} = \frac{0.125 \times 2.68}{0.52} = 0.645 \text{ or } 64.5\%$$

$$\text{Air content, } A = n(1 - S_r) = 0.34 \times 0.355$$

$$= 0.121 \text{ or } 12.1\%$$

Example 2:

The saturated unit weight, γ_{sat}, of a soil is 19.5 kN/m^3, and the specific gravity of soil solids is 2.65.

a. Derive an expression for γ_d in terms of γ_{sat}, γ_w, and G_s.
b. Using the expression derived in part (a), determine the dry unit weight of the soil.

Solution

a. From Eq. (3.19),

$$\gamma_{sat} = \frac{G_s\gamma_w + e\gamma_w}{1 + e}$$

$$\gamma_{sat} - \gamma_w = \frac{G_s\gamma_w + e\gamma_w}{1 + e} - \gamma_w = \frac{G_s\gamma_w + e\gamma_w - \gamma_w - e\gamma_w}{1 + e} = \frac{\gamma_w(G_s - 1)}{1 + e}$$

$$\gamma_{sat} - \gamma_w = \frac{\gamma_w(G_s - 1)G_s}{(1 + e)G_s} = \frac{\gamma_d(G_s - 1)}{G_s}$$

or

$$\gamma_d = \frac{(\gamma_{sat} - \gamma_w)G_s}{G_s - 1}$$

b. Given that $\gamma_{sat} = 19.5$ kN/m^3 and $G_s = 2.65$,

$$\gamma_d = \frac{(\gamma_{sat} - \gamma_w)G_s}{G_s - 1} = \frac{(19.5 - 9.81)(2.65)}{2.65 - 1} = 15.56 \text{ kN/m}^3$$

Example 3:

In its natural state, a moist soil has a volume of 0.33 ft^3 and weighs 39.93 lb. The oven-dried weight of the soil is 34.54 lb. If G_s = 2.67, calculate

a. Moisture content (%)
b. Moist unit weight (lb/ft^3)
c. Dry unit weight (lb/ft^3)
d. Void ratio
e. Porosity
f. Degree of saturation (%)

Solution

a.

$$w = \frac{W_w}{W_s} = \frac{39.93 - 34.54}{34.54}(100) = \mathbf{15.6\%}$$

b.

$$\gamma = \frac{W}{V} = \frac{39.93}{0.33} = \mathbf{121\ lb/ft^3}$$

c.

$$\gamma_d = \frac{W_s}{V} = \frac{34.54}{0.33} = \mathbf{104.7\ lb/ft^3}$$

d. The volume of solids is

$$V_s = \frac{W_s}{G_s\gamma_w} = \frac{34.54}{(2.67)(62.4)} = 0.207\ \text{ft}^3$$

Thus,

$$V_v = V - V_s = 0.33 - 0.207 = 0.123\ \text{ft}^3$$

The volume of water is

$$V_w = \frac{W_w}{\gamma_w} = \frac{39.93 - 34.54}{62.4} = 0.086\ \text{ft}^3$$

Now, refer to Figure 3.7. From Eq. (3.3),

$$e = \frac{V_v}{V_s} = \frac{0.123}{0.207} = \mathbf{0.59}$$

e.

$$n = \frac{V_v}{V} = \frac{0.123}{0.33} = \mathbf{0.37}$$

f.

$$S = \frac{V_w}{V_v} = \frac{0.086}{0.123} = 0.699 = \mathbf{69.9\%}$$

Example 4: A soil specimen is 38 mm in diameter and 76 mm long and its natural condition weighs 168 gm when dried completely in an oven the specimen weighs 130.5 gm. The value of $G_s = 2.73$. what is the degree of saturation of the specimen?

Solution: Dia = 38 mm = 3.8 cm

L= 76 mm = 7.6 cm

$$v_t = (\frac{3.8}{2})^2 \pi * 7.6 = 86.192\ cm^3$$

$$m_w = 168 - 130.5 = 37.5\ gm$$

$$v_w = \frac{37.5}{1} = 37.5\ cm^3$$

$$v_s = \frac{w_s}{G_s * \gamma_w} = \frac{130.5}{2.73 * 1} = 47.8\ cm^3$$

$$Vv = V - Vs = 86.192 - 47.8 = 38.39\ gm$$

$$S = V_w/V_v = 37.5 / 38.39 = 0.97 = 97\ \%$$

Example 5: Given mass of wet sample = 254 gm, void ratio = 0.6133, volume of air = 1.9 cm^3, mass of solid =210 gm. Determine degree of saturation, air content and dry unit weight.

Solution: mt = 254 gm, ms = 210 g $\longrightarrow m_w = 254 - 210 = 44\ gm$

$$v_w = \frac{m_w}{\rho_w} = \frac{44}{1} = 44\ cm^3$$

$$v_v = v_w + v_a = 44 + 1.9 = 45.9$$

$$0.6133 = \frac{45.9}{v_s} \rightarrow \therefore v_s = 74\ cm^3$$

$$S = \frac{v_w}{v_v} = \frac{44}{45.9} = 95.8\% \quad \rightarrow A = n(1-s) = \frac{0.6133}{1+0.6133}(1-0.95) = 0.019$$

$$\rho_{dry} = \frac{m_s}{v_t}$$

$$v_t = v_w + v_{air} + v_s = 44 + 1.9 + 74.84 = 120\ cm^3$$

$$\therefore \rho_{dry} = \frac{210}{120} = 1.75 \frac{gm}{cm^3} \rightarrow \gamma_{dry} = 17.5\ kN/m^3$$

3. PLASTICITY OF FINE SOILS

Plasticity is an important characteristic in the case of **fine soils**, the term plasticity describing the ability of a soil to undergo unrecoverable deformation without cracking or crumbling. In general, depending on its water content (defined as the ratio of the mass of water in the soil to the mass of solid particles), a soil may exist in one of the liquid, plastic, semi-solid and solid states. If the water content of a soil initially in the liquid state is gradually reduced, the state will change from liquid through plastic and semi-solid, accompanied by gradually reducing volume, until the solid state is reached. The water contents at which the transitions between states occur differ from soil to soil. In the ground, most fine soils exist in the plastic state. Plasticity is due to the presence of a significant content of clay mineral particles (or organic material) in the soil. The void space between such particles is generally very small in size with the result that water is held at negative pressure by capillary tension. This produces a degree of cohesion between the particles, allowing the soil to be deformed or moulded. Adsorption of water due to the surface forces on clay mineral particles may contribute to plastic behaviour. Any decrease in water content results in a decrease in cation layer thickness and an increase in the net attractive forces between particles. The upper and lower limits of the range of water content over which the soil exhibits plastic behaviour are defined as the liquid limit (w_L) and the plastic limit (w_P), respectively.

The water content range itself is defined as the plasticity index (I_P), i.e.:

$$I_P = w_L - w_P$$

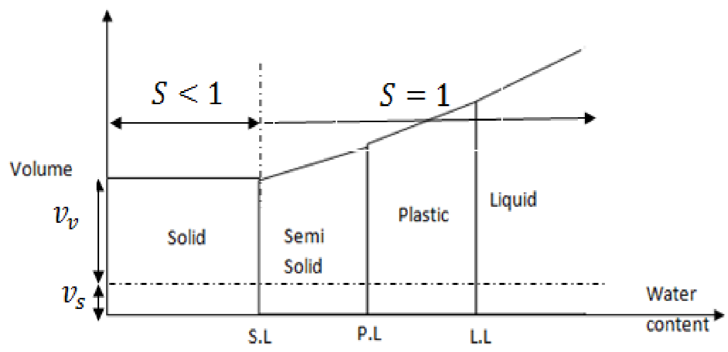
However, the transitions between the different states are gradual and the liquid and plastic limits must be defined arbitrarily. The natural water content (w) of a soil (adjusted to an equivalent water content of the fraction passing the 425-µm sieve) relative to the liquid and plastic limits can be represented by means of the liquidity index (I_L), where

$$I_L = \frac{w - w_P}{I_P}$$

The degree of plasticity of the clay-size fraction of a soil is expressed by **the ratio of the plasticity index to the percentage of clay-size particles in the soil**: this ratio is called the ***activity***.

The transition between the semi-solid and solid states occurs at the **shrinkage limit**, defined as **the water content at which the volume of the soil reaches its lowest value as it dries out**.

Atterberg, a Swedish scientist developed a method for describing the limit consistency of fine grained soils on the basis of moisture content. These limits are **liquid limit**, **plastic limit** and **shrinkage limit**.



Liquid limit (L.L): is defined as the moisture content in percent at which the soil changes from liquid to plastic state.

Plastic Limit (P.L.): The moisture contents in % at which the soil changes from plastic to semi solid state.

Shrinkage Limit (S.L.): The moisture contents in % at which the soil changes from semi solid to solid state.

Plasticity Index (P.I.): it is the range in moisture content when the soil exhibited its plastic behavior:

$$PI = LL - PL$$

Liquidity Index (L.I. or IL) : a relation between the natural moisture contents (w_n) and (L.L.) and (P.L.) in form:

$$LI = \frac{w_n - PL}{LL - PL}$$

If LI > 1 Then the soil at Liquid state
If LI = 1 then the soil at L.L.
If LI < 1 then the soil below L.L.

Activity: is the degree of plasticity of the clay size fraction of the soil and is expressed as:

$$Activity = \frac{P.I}{\%\ of\ clay\ size\ particles}$$

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Relative Density: is the ratio of the actual density to the maximum possible density of the soil it is expressed in terms of void ratio.

$$RD(\%) = \frac{e_{max} - e_n}{e_{max} - e_{min}} * 100$$

Or $$RD(\%) = \frac{\gamma_{dmax}}{\gamma_{dn}} * \frac{\gamma_{dn} - \gamma_{dmin}}{\gamma_{dmax} - \gamma_{dmin}} * 100$$

e_{max}: The void ratio of the soil in its loosest condition

e_{min}: The void ratio of the soil in its densest condition

e_n: The void ratio of the soil in its natural condition

γ_{dmax}: Maximum dry unit weight (ate_{min})

γ_{dmin}: Minimum dry unit weight (ate_{max})

γ_{dn}: Natural dry unit weight (ate_{n})

RD	Description
$0 \ - \frac{1}{3}$	loose
$\frac{1}{3} - \frac{2}{3}$	medium
$\frac{2}{3} - \ 1$	Dense

Example 1: for a granular soil, given,$\gamma_{dry} = 17.3 \frac{kN}{m^3}$, relative density = 82%, $\omega = 8\%$ and $G_s = 2.65$. If $e_{min} = 0.44$. what would be e_{max} ? what would be the dry unit weight in the loosest state?

Solution:

$$\gamma_{dry} = \frac{G_s}{1+e_n} * 10 \quad \Longrightarrow \quad 17.3 = \frac{2.65}{1+e_n} * 10$$

$$\therefore \ e_n = 0.53 \quad \Longrightarrow \quad RD = \frac{e_{max}-e_n}{e_{max}-e_{min}} * 100$$

$$0.82 = \frac{e_{max}-0.53}{e_{max}-0.44} \quad \Longrightarrow \quad \therefore \ e_{max} = 0.94$$

$$\therefore \ \gamma_{dry}\,(at\ loosest) = \frac{G_s}{1+e_{max}}\gamma_w = \frac{2.65}{1+0.94} * 10$$

$$= 13.65 \ kN/m^3$$

Example 2: a granular soil is compacted to moist unit weight of 20.45 kN/m^3 at moisture content of 18% . What is relative density of the compacted soil? Given, $e_{max} = 0.85$, $e_{min} = 0.42$ and $G_s = 2.65$?

Solution:

$$\gamma = \frac{G_s(1+\omega_c)}{1+e_n}\gamma_w \quad \Longrightarrow \quad 20.45 = \frac{2.65(1+0.18)}{1+e} * 10$$

$$\therefore e_n = 0.52 \quad \Longrightarrow \quad RD = \frac{e_{max}-e_n}{e_{max}-e_{min}} =$$

$$RD = \frac{0.85-0.52}{0.85-0.42} * 100 = 76.74\%$$

4. Soil description and classification:

It is essential that a standard language should exist for the description of soils. A comprehensive description should include the characteristics of both the soil material and the in-situ soil mass. Material characteristics can be determined from disturbed samples of the soil, i.e. samples having the same particle size distribution as the in-situ soil but in which the in-situ structure has not been preserved. The principal material characteristics are particle size distribution (or grading) and plasticity, from which the soil name can be deduced. Particle size distribution and plasticity properties can be determined either by standard laboratory tests or by simple visual and manual procedures. Secondary material characteristics are the colour of the soil and the shape, texture and composition of the particles. Mass characteristics should ideally be determined in the field but in many cases they can be detected in undisturbed samples, i.e. samples in which the in-situ soil structure has been essentially preserved. A description of mass characteristics should include an assessment of in-situ compactive state (coarse soils) or stiffness (fine soils) and details of any bedding, discontinuities and weathering. The arrangement of minor geological details, referred to as the soil macro-fabric, should be carefully described, as this can influence the engineering behaviour of the in-situ soil to a considerable extent. Examples of macro-fabric features are thin layers of fine sand and silt in clay, silt-filled fissures in clay, small lenses of clay in sand, organic inclusions and root holes. The name of the geological formation, if definitely known, should be included in the description; in addition, the type of deposit may be stated (e.g. till, alluvium, river errace), as this can indicate, in a general way, the likely behaviour of the soil. It is important to distinguish between soil description and soil classification. Soil description includes details of both material and mass characteristics, and therefore it is unlikely that any two soils will have identical descriptions. In soil classification, on the other hand, a soil is allocated to one of a limited number of groups on the basis of material characteristics only. Soil classification is thus independent of the in-situ condition of the soil mass. If the soil is to be employed in its undisturbed condition, for example to support a foundation, a full soil description will be adequate and the addition of the soil classification is discretionary. However, classification is particularly useful if the soil in question is to be used as a construction material, for example in an embankment. Engineers can also draw on past experience of the behaviour of soils of similar classification.

Basic characteristics of soils

Composite types of coarse soil	
Slightly sandy GRAVEL	Up to 5% sand
Sandy GRAVEL	5–20% sand
Very sandy GRAVEL	Over 20% sand
SAND and GRAVEL	About equal proportions
Very gravelly SAND	Over 20% gravel
Gravelly SAND	5–20% gravel
Slightly gravelly SAND	Up to 5% gravel
Slightly silty SAND (and/or GRAVEL)	Up to 5% silt
Silty SAND (and/or GRAVEL)	5–20% silt
Very silty SAND (and/or GRAVEL)	Over 20% silt
Slightly clayey SAND (and/or GRAVEL)	Up to 5% clay
Clayey SAND (and/or GRAVEL)	5–20% clay
Very clayey SAND (and/or GRAVEL)	Over 20% clay

Notes
Terms such as 'Slightly clayey gravelly SAND' (having less than 5% clay and gravel) and 'Silty sandy GRAVEL' (having 5–20% silt and sand) can be used, based on the above proportions of secondary constituents.

4.1 Grain size distribution curve

In soil mechanics, it is virtually always useful to quantify the size of the grains in a type of soil. Since a given soil will often be made up of grains of many different sizes, sizes are measured in terms of grain size distributions. Grain size distribution (GSD) information can be of value in providing initial rough estimates of a soil's engineering properties such as permeability, strength, expansivity, etc. When measuring GSDs for soils, two methods are generally used:
First for grains larger than 0.075mm sieving is used and the Second for grains in the range of 0.075mm > D > 0.5µm, the hydrometer test is used.

Sieve analysis is one of the oldest methods of size analysis. Sieve analysis is accomplished by passing a known weight of sample material successively through finer sieves and weighing the amount collected on each sieve to determine the percentage weight in each size fraction. Sieving is carried out with wet or dry materials and the sieves are usually agitated to expose all the particles to the openings. The process of sieving may be divided into two stages.

First, the elimination of particles considerably smaller than the screen apertures, which should occur fairly rapidly and, second, the separation of the so-called "near-size" particles, which is a gradual process rarely reaching final completion.

The effectiveness of a sieving test depends on the amount of material put on the sieve (the "charge") and the type of movement imparted to the sieve.

Test sieves are designated by the nominal aperture size, which is the nominal central separation of opposite sides of a square aperture or the nominal diameter of a round aperture.

The woven sieve is the oldest design, and it is normally made by weaving fine metal wire into a square pattern, then soldering the edges securely into a flattish cylindrical Container.
Woven-wire sieves were originally designated by a mesh number, which referred to the number of wires per inch, which is the same as the number of square apertures per square inch.
The table shows the standard sieve sizes used in sieve analysis test.

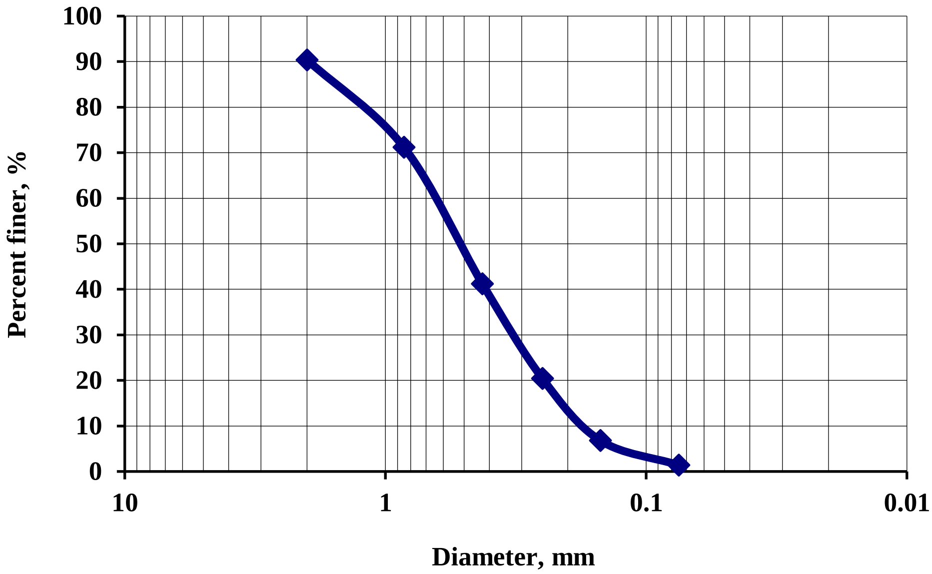
Table U.S. standard sieves

Sieve no.	Opening size, mm
3	6.35
4	4.75
6	3.36
8	2.38
10	2.00
16	1.19
20	0.84
30	0.59
40	0.425
50	0.297
60	0.25
70	0.21
100	0.149
140	0.105
200	0.075
270	0.053

There are several ways in which the results of a sieve test can be tabulated. One of these ways is shown in the table below:

	sample mass =500 gm				
Sieve No.	seive diameter (mm)	mass retained (gm)	percentage retained (%)	cumulative percentage retained (%)	percent finer (passing)
10	2	48	9.6	9.6	90.4
20	0.85	96	19.2	28.8	71.2
40	0.425	150	30	58.8	41.2
60	0.25	104	20.8	79.6	20.4
100	0.15	68	13.6	93.2	6.8
200	0.075	27	5.4	98.6	1.4
pan	0	7	1.4	100	0
		500			

The relationship between the percent finer (passing) and the diameter can be drawn using a semi-log paper as shown in the figure below:



The *effective size* of a soil is the diameter through which 10% of the total soil mass is passing and is referred to as D_{10}. The *uniformity coefficient* C_u is defined as

$$C_u = \frac{D_{60}}{D_{10}}$$

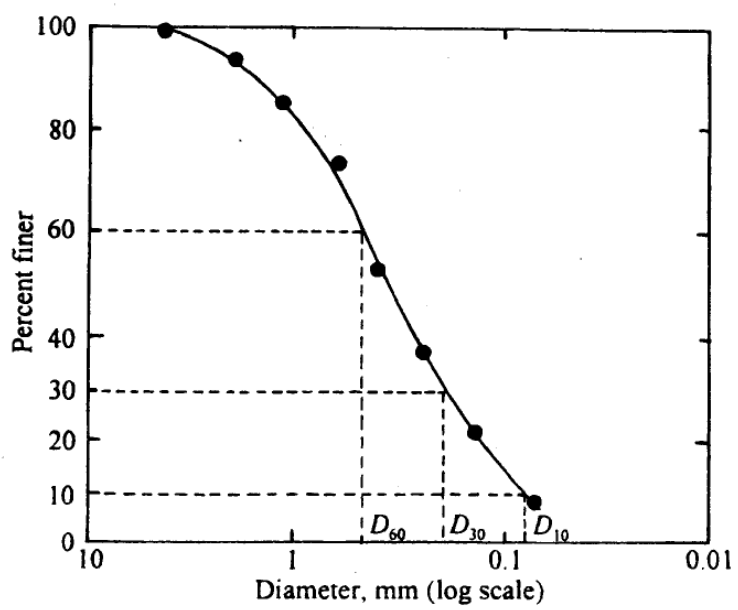
where D_{60} is the diameter through which 60% of the total soil mass is passing. The *coefficient of gradation* C_c is defined as

$$C_c = \frac{(D_{30})^2}{(D_{60})(D_{10})}$$

where D_{30} is the diameter through which 30% of the total soil mass is passing.

A soil is called a *well-graded* soil if the distribution of the grain sizes extends over a rather large range. In that case, the value of the uniformity coefficient is large. Generally, a soil is referred to as well graded if C_u is larger than about 4–6 and C_c is between 1 and 3. When most of the grains in a soil mass are of approximately the same size—i.e., C_u is close to 1—the soil is called *poorly graded*. A soil might have a combination of two or more well-graded soil fractions, and this type of soil is referred to as a *gap-graded* soil.

The sieve analysis technique described above is applicable for soil grains larger than No. 200 (0.075 mm) sieve size. For fine-grained soils the procedure used for determination of the grain-size distribution is hydrometer analysis. This is based on the principle of sedimentation of soil grains.



Grain-size distribution of a sandy soil.

4.2.1 AASHTO Classification System

The AASHTO system of soil classification was developed in 1929 as the Public Road Administration Classification System. It has undergone several revisions, with the present version proposed by the Committee on Classification of Materials for Subgrades and Granular Type Roads of the Highway Research Board in 1945 (ASTM designation D-3282; AASHTO method M145).

The AASHTO classification in present use is given in Table 4.1. According to this system, soil is classified into seven major groups: A-1 through A-7. Soils classified under groups A-1, A-2, and A-3 are granular materials of which 35% or less of the particles pass through the No. 200 sieve. Soils of which more than 35% pass through the No. 200 sieve are classified under groups A-4, A-5, A-6, and A-7. These soils are mostly silt and clay-type materials. The classification system is based on the following critiera:

1. *Grain size*
   a. *Gravel:* fraction passing the 75-mm (3-in.) sieve and retained on the No. 10 (2-mm) U.S. sieve
   b. *Sand:* fraction passing the No. 10 (2-mm) U.S. sieve and retained on the No. 200 (0.075-mm) U.S. sieve
   c. *Silt and clay:* fraction passing the No. 200 U.S. sieve

2. *Plasticity:* The term *silty* is applied when the fine fractions of the soil have a plasticity index of 10 or less. The term *clayey* is applied when the fine fractions have a plasticity index of 11 or more.
3. If cobbles and *boulders* (size larger than 75 mm) are encountered, they are excluded from the portion of the soil sample from which classification is made. However, the percentage of such material is recorded.

To classify a soil according to Table 4.1, one must apply the test data from left to right. By process of elimination, the first group from the left into which the test data fit is the correct classification. Figure 4.1 shows a plot of the range of the liquid limit and the plasticity index for soils that fall into groups A-2, A-4, A-5, A-6, and A-7.

To evaluate the quality of a soil as a highway subgrade material, one must also incorporate a number called the *group index* (GI) with the groups and subgroups of the soil. This index is written in parentheses after the group or subgroup designation. The group index is given by the equation

$$GI = (F_{200} - 35)[0.2 + 0.005(LL - 40)] + 0.01(F_{200} - 15)(PI - 10) \qquad (4.1)$$

where F_{200} = percentage passing through the No. 200 sieve
LL = liquid limit
PI = plasticity index

The first term of Eq. (4.1) — that is, $(F_{200} - 35)[0.2 + 0.005(LL - 40)]$ — is the partial group index determined from the liquid limit. The second term — that is, $0.01(F_{200} - 15)(PI - 10)$ — is the partial group index determined from the plasticity index. Following are some rules for determining the group index:

1. If Eq. (4.1) yields a negative value for GI, it is taken as 0.
2. The group index calculated from Eq. (4.1) is rounded off to the nearest whole number (for example, $GI = 3.4$ is rounded off to 3; $GI = 3.5$ is rounded off to 4).
3. There is no upper limit for the group index.
4. The group index of soils belonging to groups A-1-a, A-1-b, A-2-4, A-2-5, and A-3 is always 0.

5. When calculating the group index for soils that belong to groups A-2-6 and A-2-7, use the partial group index for *PI*, or

$$GI = 0.01(F_{200} - 15)(PI - 10) \tag{4.2}$$

In general, the quality of performance of a soil as a subgrade material is inversely proportional to the group index.

General Classification	Granular Materials 35 percent or less of total sample passing No. 200 (75 μm)								Silt-Clay Materials More than 35 percent of total sample passing No. 200 (75 μm)						
Group Classification	A-1		A-3 [1]		A-2				A-4		A-5	A-6		A-7	
	A-1-a	A-1-b	A-3	A-3a	A-2-4	A-2-5	A-2-6	A-2-7	A-4a	A-4b		A-6a	A-6b	A-7-5	A-7-6
Sieve analysis, percent passing:						*				**	*			*	
No. 10 (2 mm)	50 max														
No. 40 (425 μm)	30 max	50 max	51 min	[2]					[3]	[4]					
No. 200 (75 μm)	15 max	25 max	10 max	35 max	35 max	35 max	35 max	35 max	36 min	50 min	36 min	36 min		36 min	
Characteristics of fraction passing No. 40															
Liquid limit	—	—	Non-	—	40 max	41 min	40 max	41 min	40 max		41 min	40 max		41 min	
Plasticity index	6 max	6 max	Plastic	6 max	10 max	10 max	11 min	11 min	10 max		10 max	11 – 15	16 min	≤LL-30	>LL-30
Group Index	0						4 max		8 max		12 max	10 max	16 max	20 max	
Usual types of significant constituent materials	Stone fragments, gravel and sand		Fine sand	Sand	Silty or clayey gravel and sand				Silty soils			Clayey soils			
General rating as subgrade	Excellent to good								Good to fair						

Notes

With the test data available, the classification of a soil is found by proceeding from left to right on the chart. The first classification that the test data fits is the correct classification.

* A-2-5 is not allowed under 703.16.B. A-5 and A-7-5 is not allowed under 703.16.A. See "Natural Soil and Natural Granular Soils" (203.02.H) in this manual

** A-4b is not allowed in the top 3 feet (1.0 m) of the embankment under 203.03.A.

[1] The placing of A-3 before A-2 is necessary in the "left to right" process, and does not indicate superiority of A-3 over A-2.

[2] A-3a must contain a minimum 50 percent combined coarse and find sand sizes (passing No. 10 but retained on No. 200, between 2 mm and 75 μm).

[3] A-4a must contain less than 50 percent silt size material (between 75 μm and 5 μm).

[4] A-4b must contain 50 percent or more silt size material (between 75 μm and 5 μm).

Example 1:

The results of the particle-size analysis of a soil are as follows:

Percent passing through the No. 10 sieve = 100
Percent passing through the No. 40 sieve = 80
Percent passing through the No. 200 sieve = 58

The liquid limit and plasticity index of the minus No. 40 fraction of the soil are 30 and 10, respectively. Classify the soil by the AASHTO system.

Solution

Using Table 4.1, since 58% of the soil is passing through the No. 200 sieve, it falls under silt-clay classifications — that is, it falls under group A-4, A-5, A-6, or A-7. Proceeding from left to right, it falls under group A-4.

From Eq. (4.1),

$$GI = (F_{200} - 35)[0.2 + 0.005(LL - 40)] + 0.01(F_{200} - 15)(PI - 10)$$
$$= (58 - 35)[0.2 + 0.005(30 - 40)] + (0.01)(58 - 15)(10 - 10)$$
$$= 3.45 \simeq 3$$

So, the soil will be classified as **A-4(3)**. ■

Example 2:

Ninety-five percent of a soil passes through the No. 200 sieve and has a liquid limit of 60 and plasticity index of 40. Classify the soil by the AASHTO system.

Solution

According to Table 4.1, this soil falls under group A-7. (Proceed in a manner similar to Example 4.1.) Since

$$\underset{\substack{\uparrow \\ PI}}{40} > \underset{\substack{\uparrow \\ LL}}{60} - 30$$

this is an A-7-6 soil. Hence,

$$GI = (F_{200} - 35)[0.2 + 0.005(LL - 40)] + 0.01(F_{200} - 15)(PI - 10)$$
$$= (95 - 35)[0.2 + 0.005(60 - 40)] + (0.01)(95 - 15)(40 - 10)$$
$$= 42$$

So, the classification is **A-7-6(42)**.

Example 3:

For a soil, given

Sieve No.	Percent passing
4	90
10	76
200	34

Liquid limit = 37
Plasticity index = 12
Classify the soil by the AASHTO system.

Solution

The percentage passing through the No. 200 sieve is less than 35, so the soil is a granular material. From Table 4.1, we see that it is type A-2-6. From Eq. (4.2),

$$GI = 0.01(F_{200} - 15)(PI - 10)$$

For this soil, $F_{200} = 34$ and $PI = 12$, so

$$GI = 0.01(34 - 15)(12 - 10) = 0.38 \approx 0$$

Thus, the soil is type **A-2-6(0)**. ■

4.2.2 Unified Soil Classification System

The original form of the Unified Soil Classification System was proposed by Casagrande in 1942 during World War II for use in airfield construction undertaken by the Army Corps of Engineers. In cooperation with the U.S. Bureau of Reclamation, the Corps revised this system in 1952. At present, it is widely used by engineers (ASTM designation D-2487). In order to use the classification system, the following points must be kept in mind:

Table 3.5 Unified Soil Classification

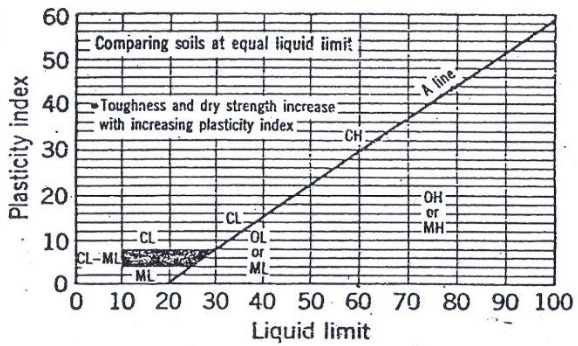
Major divisions			Field Identification Procedures (Excluding particles larger than 75 μm and basing fractions on estimated weights)	Group Symbols[a]	Typical Names	Information Required for Describing Soils	Laboratory Classification Criteria	
Coarse-grained soils More than half of material is *larger* than 75 μm sieve size[b]	Gravels More than half of coarse fraction is larger than 4 mm sieve size	Clean gravels (Little or no fines)	Wide range in grain size and substantial amounts of all intermediate particle sizes	*GW*	Well graded gravels, gravel-sand mixtures, little or no fines	Give typical name; indicate approximate percentages of sand and gravel; maximum size; angularity, surface condition, and hardness of the coarse grains; local or geologic name and other pertinent descriptive information; and symbols in parentheses	Determine percentages of gravel and sand from grain size curve. Depending on percentage of fines (fraction smaller than 75 μm sieve size) coarse grained soils are classified as follows: Less than 5% *GW, GP, SW, SP* More than 12% *GM, GC, SM, SC* 5% to 12% *Borderline* cases requiring use of dual symbols	$C_U = \frac{D_{60}}{D_{10}}$ Greater than 4 $C_C = \frac{(D_{30})^2}{D_{10} \times D_{60}}$ Between 1 and 3
			Predominantly one size or a range of sizes with some intermediate sizes missing	*GP*	Poorly graded gravels, gravel-sand mixtures, little or no fines			Not meeting all gradation requirements for *GW*
		Gravels with fines (appreciable amount of fines)	Nonplastic fines (for identification procedures see *ML* below)	*GM*	Silty gravels, poorly graded gravel-sand-silt mixtures			Atterberg limits below "A" line, or *PI* less than 4 — Above "A" line with *PI* between 4 and 7 are *borderline* cases requiring use of dual symbols
			Plastic fines (for identification procedures, see *CL* below)	*GC*	Clayey gravels, poorly graded gravel-sand-clay mixtures	For undisturbed soils add information on stratification, degree of compactness, cementation, moisture conditions and drainage characteristics		Atterberg limits above "A" line, with *PI* greater than 7
	Sands More than half of coarse fraction is smaller than 4 mm sieve size	Clean sands (Little or no fines)	Wide range in grain sizes and substantial amounts of all intermediate particle sizes	*SW*	Well graded sands, gravelly sands, little or no fines	Example: *Silty sand*, gravelly; about 20% hard, angular gravel particles 12 mm maximum size; rounded and subangular sand grains coarse to fine, about 15% non-plastic fines with low dry strength; well compacted and moist in place; alluvial sand; (*SM*)		$C_U = \frac{D_{60}}{D_{10}}$ Greater than 6 $C_C = \frac{(D_{30})^2}{D_{10} \times D_{60}}$ Between 1 and 3
			Predominantly one size or a range of sizes with some intermediate sizes missing	*SP*	Poorly graded sands, gravelly sands, little or no fines			Not meeting all gradation requirements for *SW*
		Sands with fines (appreciable amount of fines)	Nonplastic fines (for identification procedures, see *ML* below)	*SM*	Silty sands, poorly graded sand-silt mixtures			Atterberg limits below "A" line or *PI* less than 4 — Above "A" line with *PI* between 4 and 7 are *borderline* cases requiring use of dual symbols
			Plastic fines (for identification procedures, see *CL* below)	*SC*	Clayey sands, poorly graded sand-clay mixtures			Atterberg limits ~~below~~ above "A" line with *PI* greater than 7

(The 75 μm sieve size is about the smallest particle visible to naked eye)

Identification Procedures on Fraction Smaller than 380 μm Sieve Size

Major divisions		Dry Strength (crushing characteristics)	Dilatancy (reaction to shaking)	Toughness (consistency near plastic limit)	Group Symbols	Typical Names	Information Required for Describing Soils
Fine-grained soils More than half of material is *smaller* than 75 μm sieve size	Silts and clays liquid limit less than 50	None to slight	Quick to slow	None	*ML*	Inorganic silts and very fine sands, rock flour, silty or clayey fine sands with slight plasticity	Give typical name; indicate degree and character of plasticity, amount and maximum size of coarse grains; colour in wet condition, odour if any, local or geologic name, and other pertinent descriptive information, and symbol in parentheses
		Medium to high	None to very slow	Medium	*CL*	Inorganic clays of low to medium plasticity, gravelly clays, sandy clays, silty clays, lean clays	
		Slight to medium	Slow	Slight	*OL*	Organic silts and organic silt-clays of low plasticity	For undisturbed soils add information on structure, stratification, consistency in undisturbed and remoulded states, moisture and drainage conditions
	Silts and clays liquid limit greater than 50	Slight to medium	Slow to none	Slight to medium	*MH*	Inorganic silts, micaceous or diatomaceous fine sandy or silty soils, elastic silts	
		High to very high	None	High	*CH*	Inorganic clays of high plasticity, fat clays	Example: *Clayey silt*, brown; slightly plastic; small percentage of fine sand; numerous vertical root holes; firm and dry in place; loess; (*ML*)
		Medium to high	None to very slow	Slight to medium	*OH*	Organic clays of medium to high plasticity	
Highly Organic Soils		Readily identified by colour, odour, spongy feel and frequently by fibrous texture			*Pt*	Peat and other highly organic soils	

Use grain size curve in identifying the fractions as given under field identification



Plasticity chart
for laboratory classification of fine grained soils

From Wagner, 1957.

[a] *Boundary classifications*. Soils possessing characteristics of two groups are designated by combinations of group symbols. For example *GW–GC*, well graded gravel-sand mixture with clay binder.

[b] All sieve sizes on this chart are U.S. standard.

Field Identification Procedure for Fine Grained Soils or Fractions

Figure 4.6 gives the grain-size distribution of two soils. The liquid and plastic limits of minus No. 40 sieve fraction of the soil are as follows:

	Soil A	Soil B
Liquid limit	30	26
Plastic limit	22	20

Determine the group symbols and group names according to the Unified Soil Classification System.

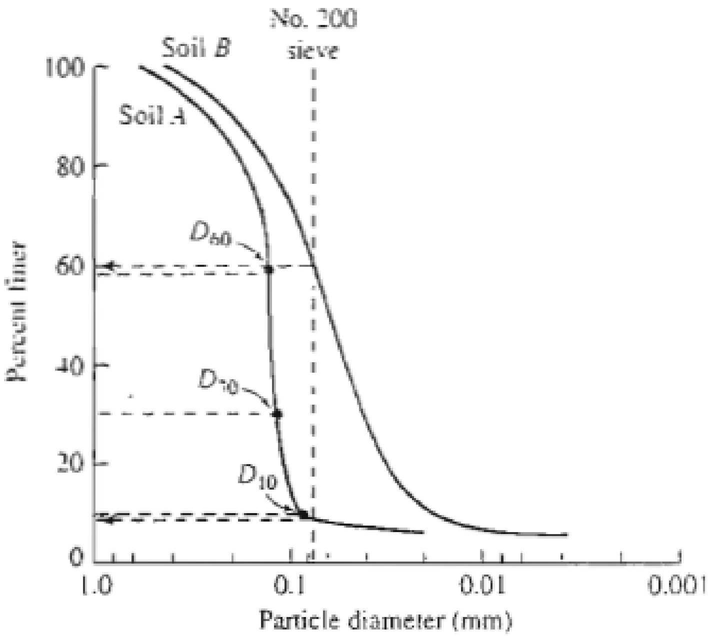


Figure 4.6 Particle-size distribution of two soils

Solution:

Soil A: 1) Percentage of passing No. 200 Sieve is 8% which is < 50%, then the soil within the upper part of classification table.

2) Percentage of passing 4mm is 100% which is > 50%, and not meeting all gradation requirements for *SW* because:

$$C_u = \frac{D_{60}}{D_{10}} = \frac{0.135}{0.085} = 1.59 < 6$$

$$C_C = \frac{D_{30}^2}{D_{60} \times D_{10}} = \frac{(0.12)^2}{(0.135)(0.085)} = 1.25 > 1$$

That means the soil is *SP* but:

3) passing No. 200 is $5 \leq 8 \leq 12$ which is lies in the borderline, that means the soil classification is dual symbols. LL and PI decide the second symbol.
LL = 30, PI = 30–22 = 8 which is lies above A-line, then the second symbol is *SC*

The final classification is *SP-SC* (Poorly graded sand with clay).

Soil B: 1) Percentage of passing No. 200 Sieve is 61% which is > 50%, then the soil within the lower part of classification table.

2) LL = 26, PI = 26–20 = 6 which is lies in the hatched area.
The final classification is *CL-ML*.

يستخدم الشكل أدناه لمعرفة تصنيف التربة الناعمة وحسب الحدود المبينة في الشكل فمثلا في المثال وقعت المواصفات ضمن المنطقة المظللة فصنفت على أنها CL-ML و بنفس الطريقة لبقية الحدود فلو كان LL=70 و PI=20 فان التربة تصنف على انها MH or OH و لو كان LL=40 و PI=20 فان التربة تصنف على انها CL or OL وهكذا و لمعرفة التربة من بين هذين الرمزين بدقة يمكن الرجوع لأغلب مصادر ميكانيك التربة فيها تفصيلات أكثر لا مجال للخوض فيها ضمن هذه المرحلة.

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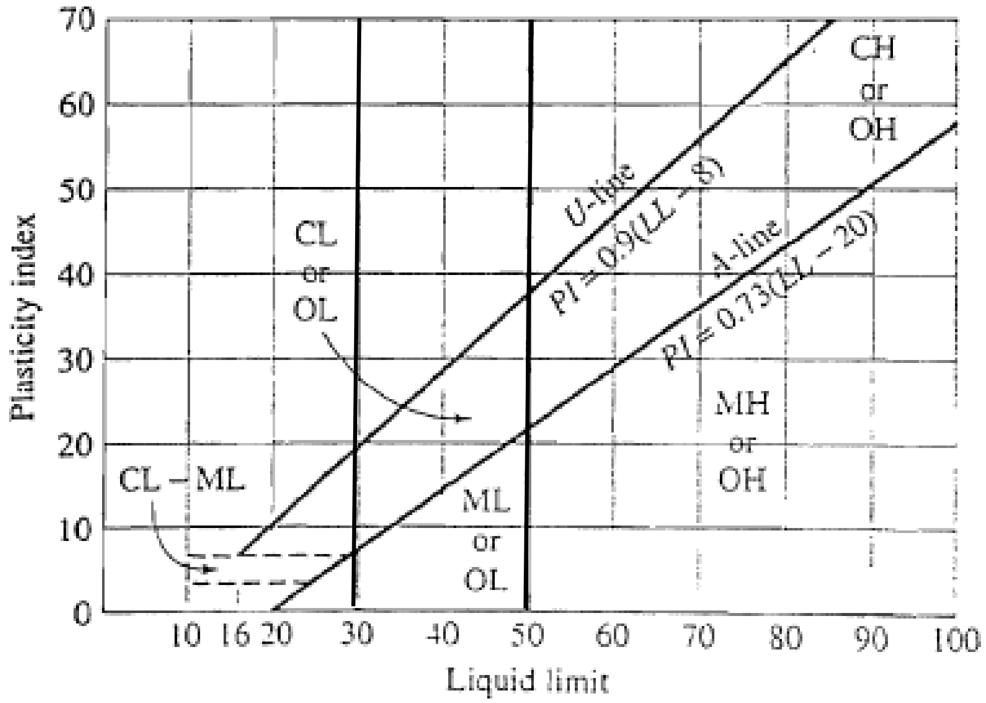


Figure 4.2 Plasticity chart

4.3 Soil compaction

Compaction is the process of increasing the density of a soil by packing the particles closer together with a reduction in the volume of *air*; there is no significant change in the volume of water in the soil. In the construction of fills and embankments, loose soil is placed in layers ranging between 75 and 450 mm in thickness, each layer being compacted to a specified standard by means of rollers, vibrators or rammers. In general, the higher the degree of compaction the higher will be the shear strength and the lower will be the compressibility of the soil. An *engineered fill* is one in which the soil has been selected, placed and compacted to an appropriate specification with the object of achieving a particular engineering performance, generally based on past experience. The aim is to ensure that the resulting fill possesses properties that are adequate for the function of the fill. This is in contrast to non-engineered fills which have been placed without regard to a subsequent engineering function.

The degree of compaction of a soil is measured in terms of dry density, i.e. the mass of solids only per unit volume of soil. If the bulk density of the soil is ρ and the water content w, then from Equations 1.17 and 1.20 it is apparent that the dry density is given by

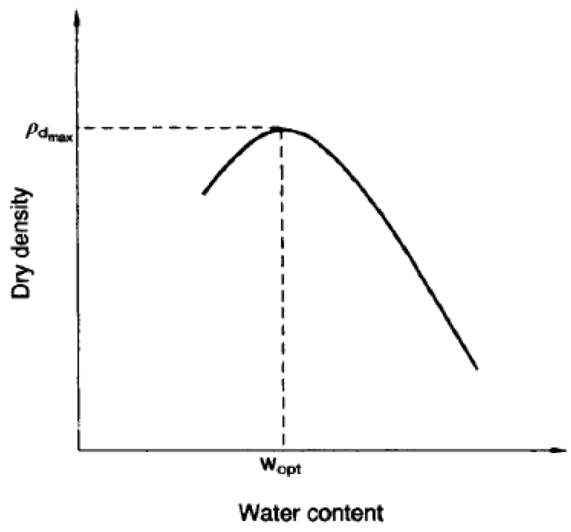
$$\rho_d = \frac{\rho}{1 + w}$$

The dry density of a given soil after compaction depends on the water content and the energy supplied by the compaction equipment (referred to as the *compactive effort*).

The compaction characteristics of a soil can be assessed by means of standard laboratory tests. The soil is compacted in a cylindrical mould using a standard compactive effort. In BS 1377 (Part 4) [2] three compaction procedures are detailed. In the *Proctor* test the volume of the mould is 1l and the soil (with all particles larger than 20 mm removed) is compacted by a rammer consisting of a 2.5-kg mass falling freely through 300 mm: the soil is compacted in three equal layers, each layer receiving 27 blows with the rammer. In the *modified AASHTO* test the mould is the same as is used in the above test but the rammer consists of a 4.5-kg mass falling 450 mm: the soil (with all particles larger than 20 mm removed) is compacted in five layers, each layer receiving 27 blows with the rammer. If the sample contains a limited proportion of particles up to 37.5 mm in size, a 2.3-l mould should be used, each layer receiving 62 blows with either the 2.5- or 4.5-kg rammer. In the *vibrating hammer* test, the soil (with all particles larger than 37.5 mm removed) is compacted in three layers in a 2.3-l mould, using a circular tamper fitted in the vibrating hammer, each layer being compacted for a period of 60 s.

After compaction using one of the three standard methods, the bulk density and water content of the soil are determined and the dry density calculated. For a given soil the process is repeated at least five times, the water content of the sample being increased each time. Dry density is plotted against water content and a curve of the form shown in Figure 1.11 is obtained. This curve shows that for a particular method of compaction (i.e. a particular compactive effort) there is a particular value of water content, known as the *optimum water content* (w_{opt}), at which a maximum value of dry density is obtained. At low values of water content most soils tend to be stiff and are difficult to compact. As the water content is increased the soil becomes more workable, facilitating compaction and resulting in higher dry densities. At high water contents, however, the dry density decreases with increasing water content, an increasing proportion of the soil volume being occupied by water.

If all the air in a soil could be expelled by compaction the soil would be in a state of full saturation and the dry density would be the maximum possible value for the given



Dry density–water content relationship.

water content. However, this degree of compaction is unattainable in practice. The maximum possible value of dry density is referred to as the 'zero air voids' dry density or the saturation dry density and can be calculated from the expression:

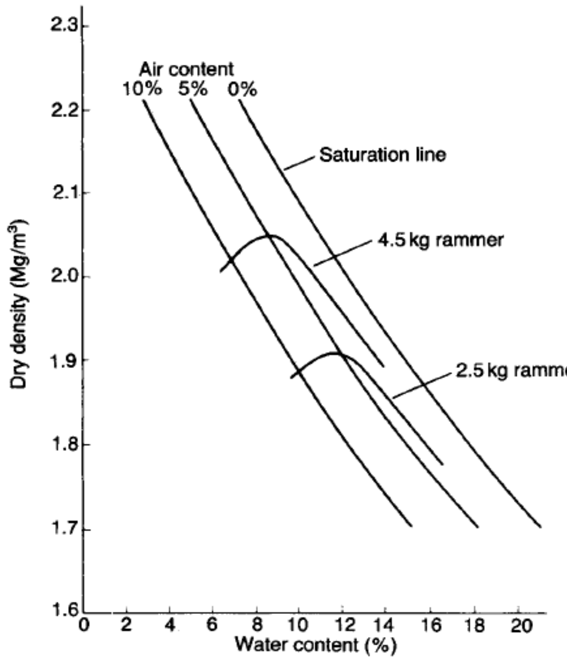
$$\rho_d = \frac{G_s}{1 + wG_s}\rho_w \tag{1.25}$$

In general, the dry density after compaction at water content w to an air content A can be calculated from the following expression, derived from Equations 1.15 and 1.20:

$$\rho_d = \frac{G_s(1 - A)}{1 + wG_s}\rho_w \tag{1.26}$$

The calculated relationship between zero air voids dry density and water content (for $G_s = 2.65$) is shown in Figure 1.12; the curve is referred to as the zero air voids line or the saturation line. The experimental dry density–water content curve for a particular compactive effort must lie completely to the left of the zero air voids line. The curves relating dry density at air contents of 5 and 10% with water content are also shown in Figure 1.12, the values of dry density being calculated from Equation 1.26. These curves enable the air content at any point on the experimental dry density–water content curve to be determined by inspection.

For a particular soil, different dry density–water content curves are obtained for different compactive efforts. Curves representing the results of tests using the 2.5- and 4.5-kg rammers are shown in Figure 1.12. The curve for the 4.5-kg test is situated above and to the left of the curve for the 2.5-kg test. Thus, a higher compactive effort results in a higher value of maximum dry density and a lower value of optimum water content; however, the values of air content at maximum dry density are approximately equal.

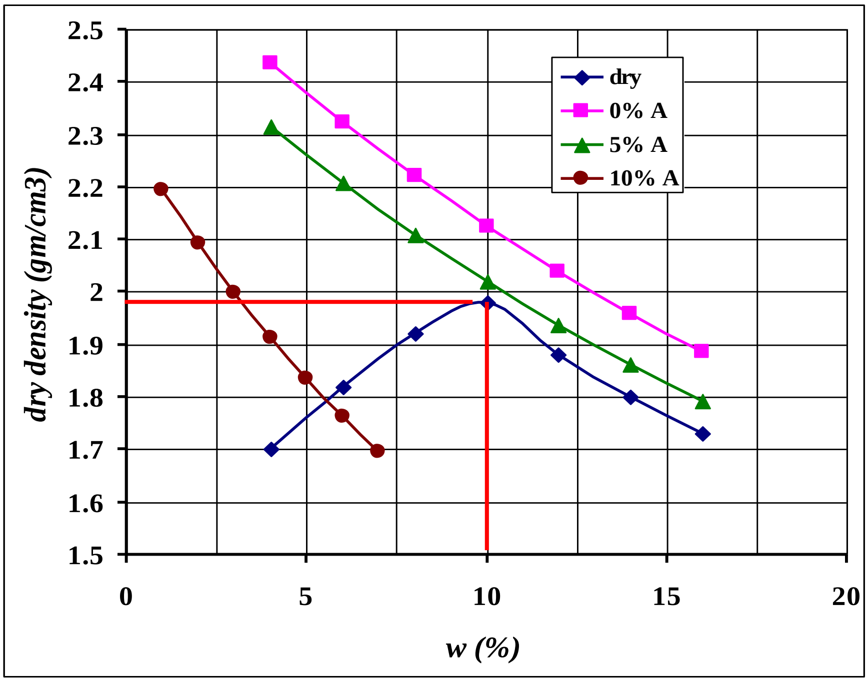


Dry density–water content curves for different compactive efforts.

The dry density–water content curves for a range of soil types using the same compactive effort (the BS 2.5-kg rammer) are shown in Figure 1.13. In general, coarse soils can be compacted to higher dry densities than fine soils.

Example: The following results were obtained from a standard compaction test. Determine the Optimum moisture content and maximum dry density. Plot the curves of 0%, 5% and 10% air content and gives the value of air content at the maximum dry density. Given the volume of standard mold is 1000 cm^3 and G_s=2.7.

Mass (gm)	w (%)	ρ	ρ_d	ρ_d for 0% Air voids	ρ_d for 5% Air voids	ρ_d for 10% Air voids
1768	4	1.77	1.7	2.44	2.31	2.19
1929	6	1.93	1.82	2.32	2.2	2.09
2074	8	2.07	1.92	2.22	2.1	1.99
2178	10	2.18	1.98	2.13	2	1.91
2106	12	2.11	1.88	2.04	1.94	1.84
2052	14	2.05	1.8	1.95	1.86	1.76
2007	16	2	1.73	1.89	1.79	1.69



Maximum dry density = 1.98 *gm/cm³*
Optimum moisture content = 10%

$$\rho_d = \frac{G_s(1-A)}{1+wG_s}\rho_w$$

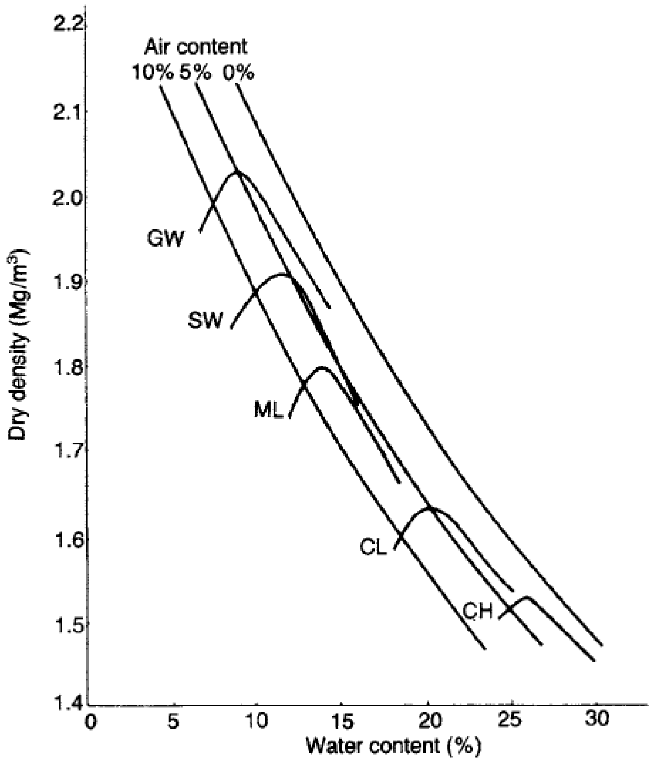
$$1.98 = \frac{2.7(1-A)}{1+0.1\times 2.7}\times 1 \qquad A = 6.8\%$$

4.4 Field compaction

The results of laboratory compaction tests are not directly applicable to field compaction because the compactive efforts in the laboratory tests are different, and are applied in a different way, from those produced by field equipment. Further, the laboratory tests are carried out only on material smaller than either 20 or 37.5 mm. However, the maximum dry densities obtained in the laboratory using the 2.5- and 4.5-kg rammers cover the range of dry density normally produced by field compaction equipment.

A minimum number of passes must be made with the chosen compaction equipment to produce the required value of dry density. This number, which depends on the type and mass of the equipment and on the thickness of the soil layer, is usually within the range 3–12. Above a certain number of passes no significant increase in dry density is obtained. In general, the thicker the soil layer the heavier the equipment required to produce an adequate degree of compaction.

There are two approaches to the achievement of a satisfactory standard of compaction in the field, known as *method* and *end-product* compaction. In method compaction



Dry density–water content curves for a range of soil types.

the type and mass of equipment, the layer depth and the number of passes are specified. In the UK these details are given, for the class of material in question, in the Specification for Highway Works [6]. In end-product compaction the required dry density is specified: the dry density of the compacted fill must be equal to or greater than a stated percentage of the maximum dry density obtained in one of the standard laboratory compaction tests. Method compaction is used in most earthworks. End-product compaction is normally restricted to pulverized fuel ash in general fill and to certain selected fills.

Field density tests can be carried out, if considered necessary, to verify the standard of compaction in earthworks, dry density or air content being calculated from measured values of bulk density and water content. A number of methods of measuring bulk density in the field are detailed in BS 1377 (Part 4) [2].

The following types of compaction equipment are used in the field.

Pulverized fuel ash
Kneading towed propelled kentledge

Smooth-wheeled rollers

These consist of hollow steel drums, the mass of which can be increased by water or sand ballast. They are suitable for most types of soil except uniform sands and silty sands, provided a mixing or kneading action is not required. A smooth surface is produced on the compacted layer, encouraging the run-off of any rainfall but resulting in relatively poor bonding between successive layers; the fill as a whole will therefore tend to be laminated. Smooth-wheeled rollers, and the other types of roller described below, can be either towed or self-propelled.

Pneumatic-tyred rollers

This equipment is suitable for a wide range of coarse and fine soils but not for uniformly graded material. Wheels are mounted close together on two axles, the rear set overlapping the lines of the front set to ensure complete coverage of the soil surface. The tyres are relatively wide with a flat tread so that the soil is not displaced laterally. This type of roller is also available with a special axle which allows the wheels to wobble, thus preventing the bridging over of low spots. Pneumatic-tyred rollers impart a kneading action to the soil. The finished surface is relatively smooth, resulting in a low degree of bonding between layers. If good bonding is essential, the compacted surface must be scarified between layers. Increased compactive effort can be obtained by increasing the tyre inflation pressure or, less effectively, by adding kentledge to the body of the roller.

Ballasting

Sheepsfoot rollers

This type of roller consists of hollow steel drums with numerous tapered or club-shaped feet projecting from their surfaces. The mass of the drums can be increased by ballasting. The arrangement of the feet can vary but they are usually from 200 to 250 mm in length with an end area of 40–65 cm^2. The feet thus impart a relatively high pressure over a small area. Initially, when the layer of soil is loose, the drums are in contact with the soil surface. Subsequently, as the projecting feet compact below the surface and the soil becomes sufficiently dense to support the high contact pressure, the drums rise above the soil. Sheepsfoot rollers are most suitable for fine soils, both plastic and non-plastic, especially at water contents dry of optimum. They are also suitable for coarse soils with more than 20% of fines. The action of the feet causes significant mixing of the soil, improving its degree of homogeneity, and will break up lumps of stiff material. Due to the penetration of the feet, excellent bonding is produced between successive soil layers, an important requirement for water-retaining earthworks. *Tamping* rollers are similar to sheepsfoot rollers but the feet have a larger end area, usually over 100 cm^2, and the total area of the feet exceeds 15% of the surface area of the drums.

Grid rollers

These rollers have a surface consisting of a network of steel bars forming a grid with square holes. Kentledge can be added to the body of the roller. Grid rollers provide high contact pressure but little kneading action and are suitable for most coarse soils.

Vibratory rollers

These are smooth-wheeled rollers fitted with a power-driven vibration mechanism. They are used for most soil types and are more efficient if the water content of the soil is slightly wet of optimum. They are particularly effective for coarse soils with little or no fines. The mass of the roller and the frequency of vibration must be matched to the soil type and layer thickness. The lower the speed of the roller the fewer the number of passes required.

Vibrating plates

This equipment, which is suitable for most soil types, consists of a steel plate with upturned edges, or a curved plate, on which a vibrator is mounted. The unit, under manual guidance, propels itself slowly over the surface of the soil.

Unit Two
Seepage

1. Soil Water

All soils are permeable materials, water being free to flow through the interconnected pores between the solid particles. The pressure of the pore water is measured relative to atmospheric pressure and the level at which the pressure is atmospheric (i.e. zero) is defined as the water table (WT) or the phreatic surface. Below the water table the soil is assumed to be fully saturated, although it is likely that, due to the presence of small volumes of entrapped air, the degree of saturation will be marginally below 100%.

Below the water table the pore water may be static, the hydrostatic pressure depending on the depth below the water table, or may be seeping through the soil under hydraulic gradient: this chapter is concerned with the second case. Bernoulli's theorem applies to the pore water but seepage velocities in soils are normally so small that velocity head can be neglected. Thus

$$h = \frac{u}{\gamma_w} + z$$

where h is the total head, u the pore water pressure, γ_w the unit weight of water (9.8 kN/m^3) and z the elevation head above a chosen datum.

In one dimension, water flows through a fully saturated soil in accordance with Darcy's empirical law:

$$q = Aki \qquad (2.2)$$

or

$$v = \frac{q}{A} = ki$$

where q is the volume of water flowing per unit time, A the cross-sectional area of soil corresponding to the flow q, k the coefficient of permeability, i the hydraulic gradient and v the discharge velocity. The units of the coefficient of permeability are those of velocity (m/s).

The coefficient of permeability depends primarily on the average size of the pores, which in turn is related to the distribution of particle sizes, particle shape and soil structure. In general, the smaller the particles the smaller is the average size of the pores and the lower is the coefficient of permeability. The presence of a small percentage of fines in a coarse-grained soil results in a value of k significantly lower than the value for the same soil without fines. For a given soil the coefficient of permeability is a function of void ratio. If a soil deposit is stratified the permeability for flow parallel to the direction of stratification is higher than that for flow perpendicular to the direction of stratification. The presence of fissures in a clay results in a much higher value of permeability compared with that of the unfissured material.

Table 2.1 Coefficient of permeability (m/s) (BS 8004: 1986)

1	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}	10^{-8}	10^{-9}	10^{-10}
Clean gravels		Clean sands and sand–gravel mixtures			Very fine sands, silts and clay-silt laminate			Unfissured clays and clay-silts (>20% clay)		
		Desiccated and fissured clays								

2. Determination of Coefficient of Permeability Soil Water (laboratory test)

The coefficient of permeability for coarse soils can be determined by means of the *constant-head* permeability test (Figure 2.1(a)). The soil specimen, at the appropriate density, is contained in a Perspex cylinder of cross-sectional area A: the specimen rests on a coarse filter or a wire mesh. A steady vertical flow of water, under a constant total head, is maintained through the soil and the volume of water flowing per unit time (q)

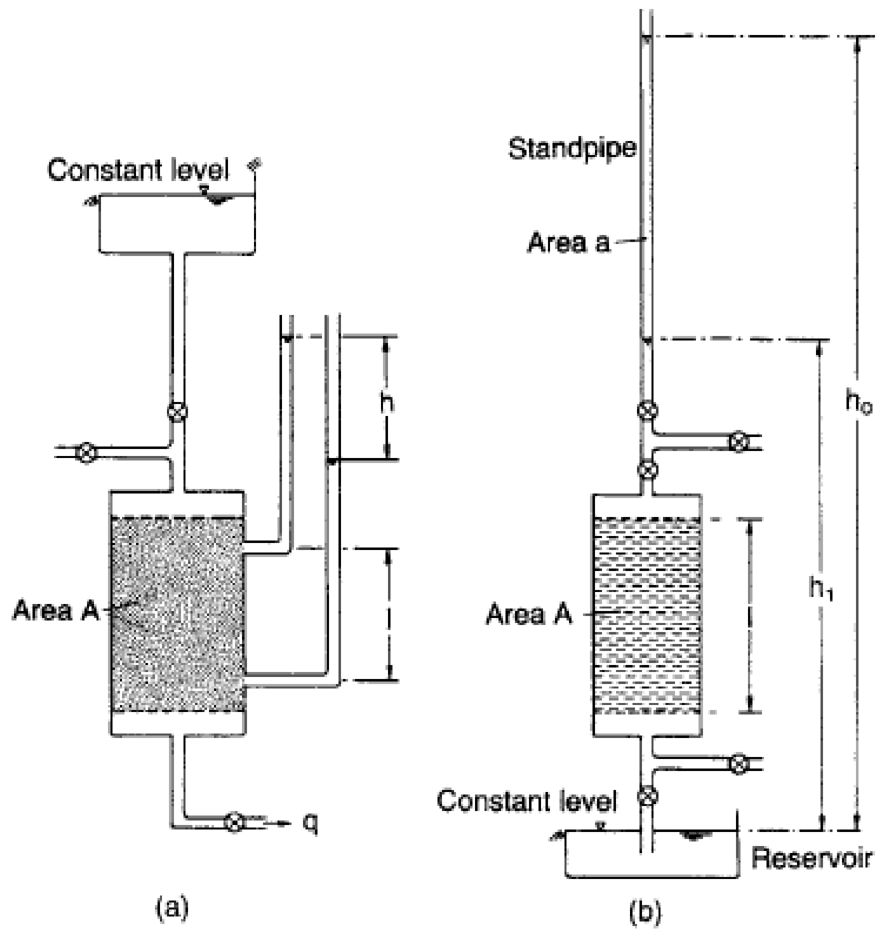


Figure 2.1 Laboratory permeability tests: (a) constant head and (b) falling head.

is measured. Tappings from the side of the cylinder enable the hydraulic gradient (h/l) to be measured. Then from Darcy's law:

$$k = \frac{ql}{Ah}$$

A series of tests should be run, each at a different rate of flow. Prior to running the test a vacuum is applied to the specimen to ensure that the degree of saturation under flow will be close to 100%. If a high degree of saturation is to be maintained the water used in the test should be de-aired.

For fine soils the *falling-head* test (Figure 2.1(b)) should be used. In the case of fine soils, undisturbed specimens are normally tested and the containing cylinder in the test may be the sampling tube itself. The length of the specimen is l and the cross-sectional area A. A coarse filter is placed at each end of the specimen and a standpipe of internal area a is connected to the top of the cylinder. The water drains into a reservoir of constant level. The standpipe is filled with water and a measurement is made of the time (t_1) for the water level (relative to the water level in the reservoir) to fall from h_0 to h_1. At any intermediate time t the water level in the standpipe is given by h and its rate of change by $-\mathrm{d}h/\mathrm{d}t$. At time t the difference in total head between the top and bottom of the specimen is h. Then, applying Darcy's law:

$$-a\frac{\mathrm{d}h}{\mathrm{d}t} = Ak\frac{h}{l}$$

$$\therefore \quad -a\int_{h_0}^{h_1}\frac{\mathrm{d}h}{h} = \frac{Ak}{l}\int_0^{t_1}\mathrm{d}t$$

$$\therefore k = \frac{al}{At_1}\ln\frac{h_0}{h_1}$$

$$= 2.3\frac{al}{At_1}\log\frac{h_0}{h_1}$$

Again, precautions must be taken to ensure that the degree of saturation remains close to 100%. A series of tests should be run using different values of h_0 and h_1 and/or standpipes of different diameters.

The coefficient of permeability of fine soils can also be determined indirectly from the results of consolidation tests (see Chapter 7).

The reliability of laboratory methods depends on the extent to which the test specimens are representative of the soil mass as a whole. More reliable results can generally be obtained by the *in-situ* methods described below.

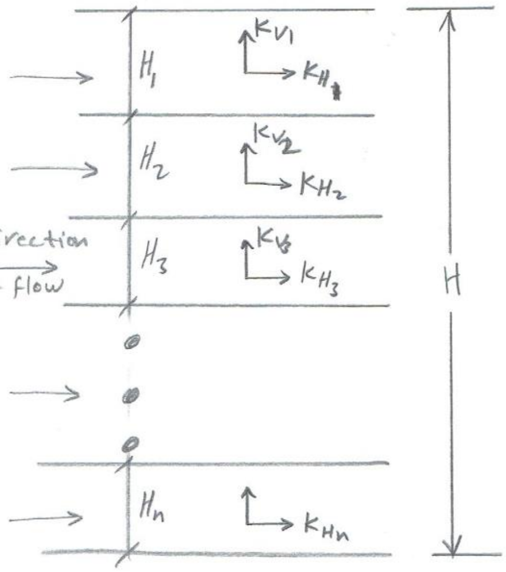
- Equivalent Permeability in stratified soil

$$q = v \cdot 1 \cdot H = v_1 \cdot 1 \cdot H_1 + v_2 \cdot 1 \cdot H_2 + v_3 \cdot 1 \cdot H_3 + \cdots + v_n \cdot 1 \cdot H_n \quad (4-31)$$

$\therefore$

$$K_{v(eq)} = \frac{H}{\left(\frac{H_1}{K_{v_1}}\right) + \left(\frac{H_2}{K_{v_2}}\right) + \left(\frac{H_3}{K_{v_3}}\right) + \cdots + \left(\frac{H_n}{K_{v_n}}\right)} \quad (4-37)$$

$$K_{H(eq)} = \frac{1}{H}\left(K_{H_1} H_1 + K_{H_2} H_2 + K_{H_3} H_3 + \cdots K_{H_n} H_n\right) \quad (4-32).$$



Ex1 : The results of a constant head permeability test for a fine sand sample having a diameter of 150 mm and a length of 300 mm are as follows

constant head difference = 500 mm.
Time of collection of water = 5 min.
Volume of water collected = 350 cc.

- Find the coefficient of permeability in $\frac{cm}{sec}$

Sol

$$K = \frac{VL}{Aht}$$

Given $Q = 350$ cc ; $L = 300$ mm ; $A = (\frac{\pi}{4})(150)^2 = 17671.46 \text{ mm}^2$,

$h = 500$ mm ; $t = 5 * 60 = 300$ sec.

So;

change to mm³

$$K = \frac{(350 * 1000) * 300}{17671.46 * 500 * 300} = 3.96 * 10^{-2} \text{ mm/sec}$$

$$= 3.96 * 10^{-3} \text{ cm/sec}$$

Ex2: Find the rate of flow of water through the permeable soil layer shown in Fig. 4-15

Solution: The hydraulic gradient (i):

$$i = \frac{4\,m}{(50\,m/\cos 8^\circ)} = 0.0792$$

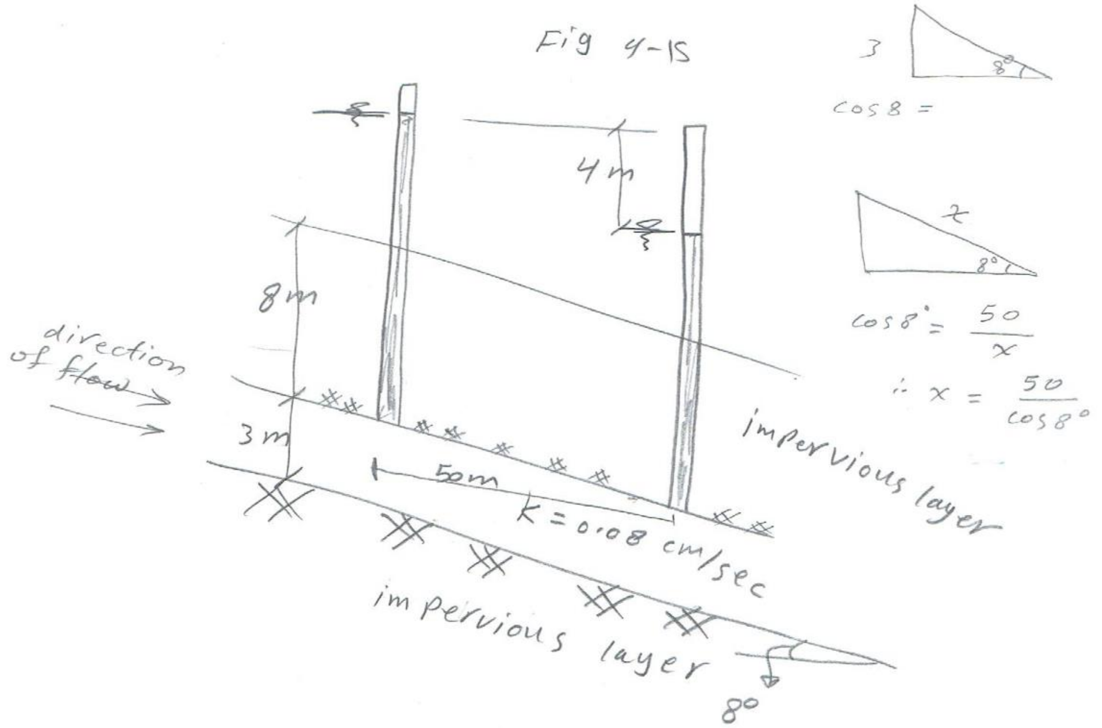
The rate of flow per meter width of the profile shown (q)

$$q = K i A = \left(\frac{0.08\ cm/sec}{10^2}\right) * 0.0792 * (3 \cos 8^\circ * 1)$$

change to m/sec

$= 0.188 * 10^{-3}$ $m^3/sec/m$ width $= 0.19 * 10^{-3}$ $m^3/sec/m$ width

Fig 4-15



Ex : A Falling-head permeability test is performed on a fine-grained soil. The soil sample has a length of 12 cm and a cross-sectional area of 6 cm². The water in the stand pipe flowing into the soil is 60 cm above the top of the sample at the start of the test. It falls 5 cm in 30 minutes. The standpipe has a cross-sectional area of 2 cm².

a) make a sketch of the described conditions.

b) what is the coefficient of permeability in centimeters per second?

c) what is the coefficient of permeability in feet per minute?

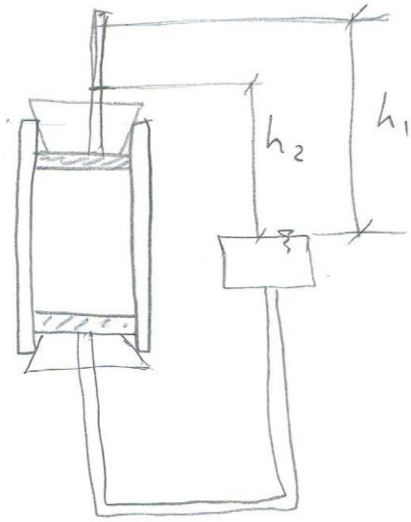
Sol

$$K = 2.303 \frac{aL}{At} \log_{10} \frac{h_1}{h_2}$$

$a = 2cm^2$; $L = 12$ cm ; $A = 6cm^2$

$t = 30 \text{ min} = 30 * 60$
$= 1800$ sec.

$h_1 = 60$ cm , $h_2 = 60 - 5$
$= 55$ cm



$$\therefore K = 2.303 \frac{2 cm^2 * 12 cm}{6 cm^2 * 1800 sec} \log \frac{60}{55}$$

$= 0.0001933$ cm/sec

$1.93 * 10^{-4} \frac{cm}{sec} \div (2.54 * 12) \frac{ft}{cm}$

$= 6.34 * 10^{-6} \frac{ft}{sec} * 60 \frac{sec}{min} = 3.8 * 10^{-4} \frac{ft}{min}$.

3. Flow Net (Two Dimensional Flow)

- Flow line :- is a line along which a water particle will travel from upstream to downstream side in the permeable soil medium.

- equipotential line : is a line along which the potential head at all points is the same. thus, if piezometers are placed at different points along an equipotential line, the height of water will rise to the same elevation in all of them.

Figure 4-17a

- Flow net :- is a combination of a number of flow lines and equipotential lines.

notes

1- The equipotential lines intersect the flow lines at right angles.

2- The flow elements formed are aproximate squares.

Figure 4-17b – Figure 4-18

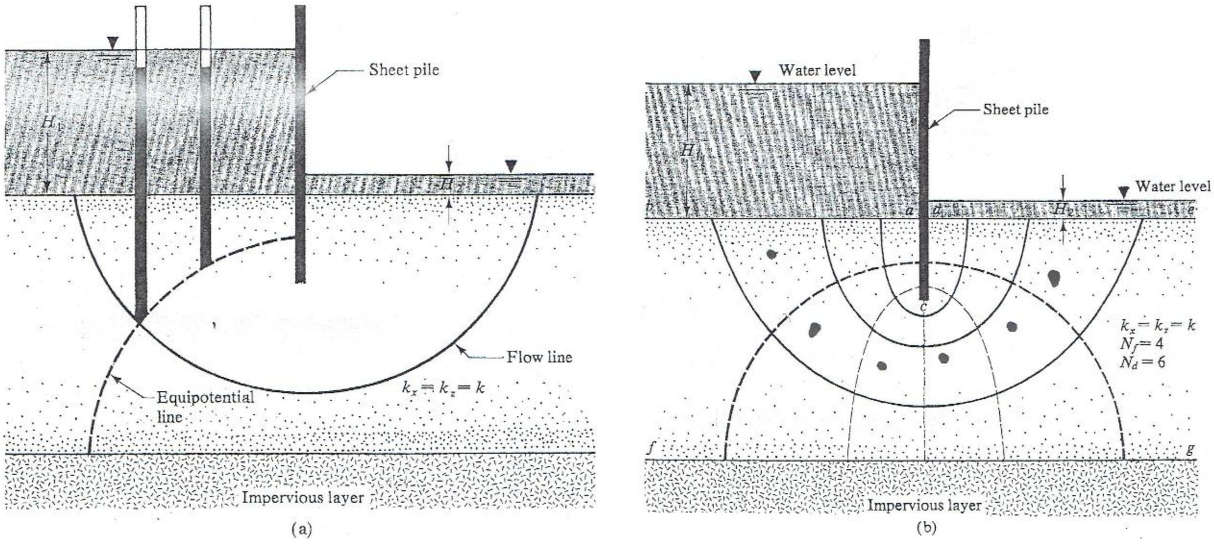


Figure 4.17 (a) Definition of flow lines and equipotential lines; (b) completed flow net

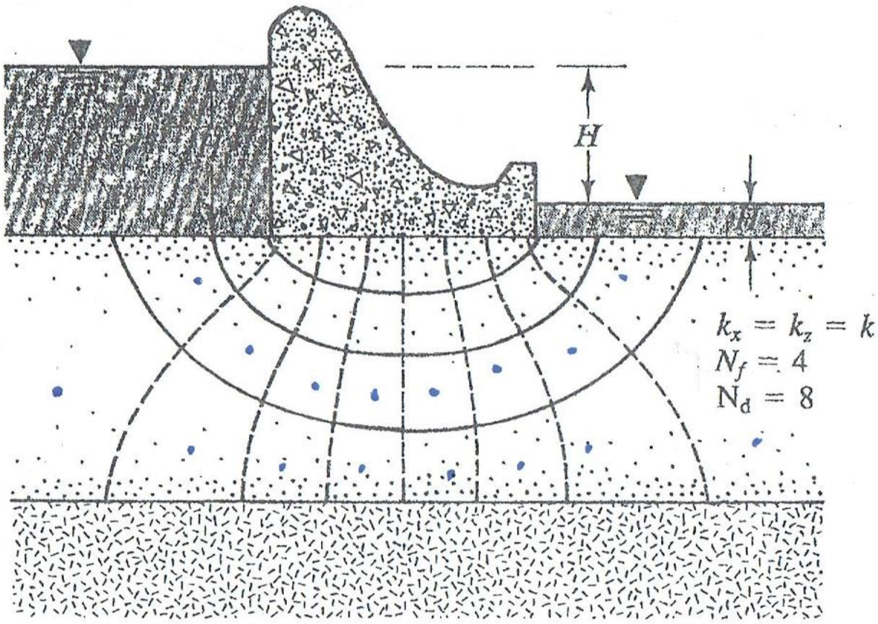


Figure 4.18 Flow net under a dam

* seepage calculation from a flow net.

$$q = K H \frac{N_f}{N_d} \longrightarrow (4-51)$$

where ;

q = The rate of seepage through the flow channel per unit width.

K = coefficient of permeability ($K_x = K_z = K$).

N_f = No. of flow channel

N_d = No. of potential drops (Figures).

H = the difference of head between upstream and downstream sides.

- Flow net in Anisotropic soil

$$q = \sqrt{k_x \cdot k_z}\ \frac{H N_f}{N_d}$$

$$\Delta q = k \frac{H}{N_d}$$

H = the difference of head between the upstream and downstream sides

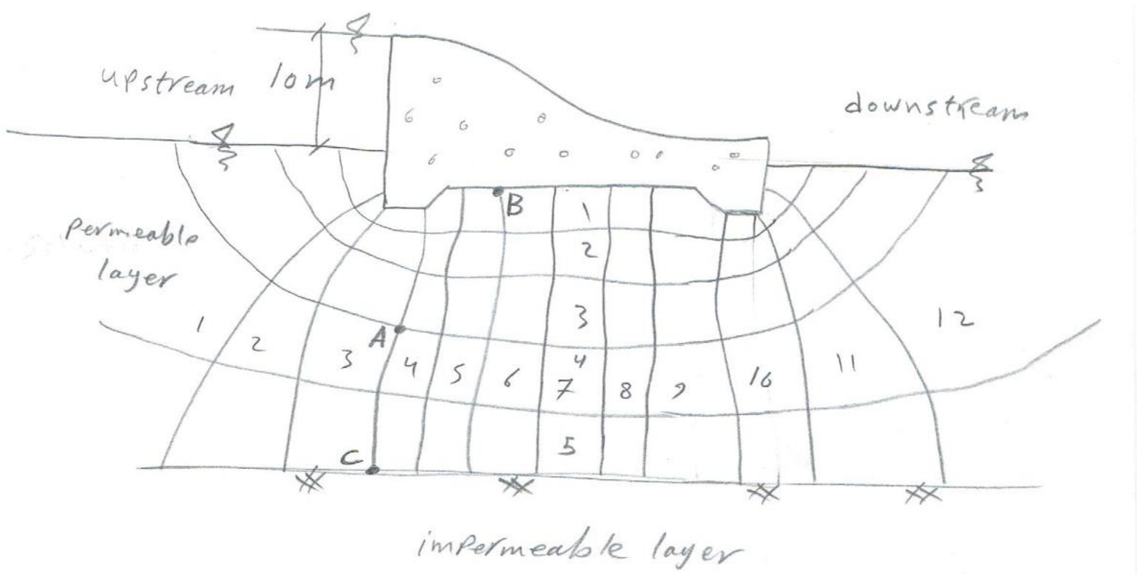
N_d = number of potential drops.

Ex: For the flow net shown in Fig.

a) How high would water rise if a piezometer is placed at (i) A, (ii) B, (iii) C ?

b) If k = 0.01 mm/sec. determine the seepage loss of the dam in m³/(day.m).

c) draw uplift distribution and calculate the uplift force on the structure.



Sol.

The maximum hydraulic head h is 10m.
in Fig. $N_d = 12$

$\therefore \Delta h = h / N_d = 10/12 = 0.833\ m$

Part (a); (i); To reach A, water has to go through three potential drops, So head lost is equal to

$3 * 0.833 = 2.5\ m$; Hence; the elevation of the water level in the piezometer at A will be $10\ m - 2.5\ m = 7.5\ m$ above the ground surface.

Part (a); (ii): The water level in the piezometer at (B) $= 10 - \underbrace{5}_{\text{No. of drops}} (0.833) = \underline{5.84}\ m$

Part (a); (iii)

at C $= 10 - 3(0.833) = \underline{7.5\ m}$ as same as A
because it falls on the same equipotential line.

Part (B)

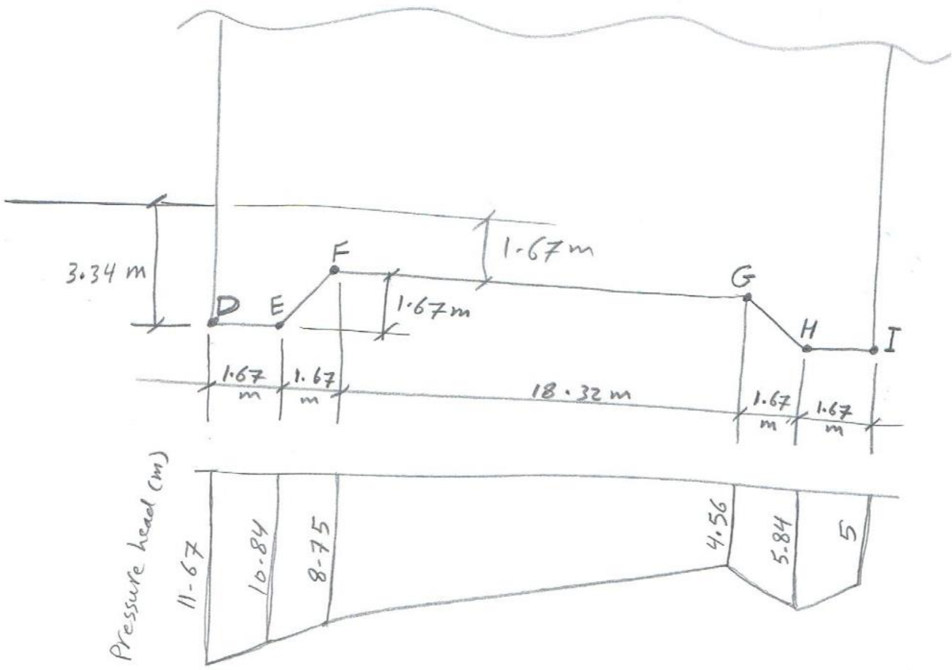
The seepage loss is given by

$q = KH\left(\frac{N_F}{N_d}\right) \Rightarrow N_f = 5\ ;\ N_d = 12$

$\therefore K = 0.01\ mm/sec = \left(\frac{0.01\ mm/sec}{1000\ \frac{mm}{m}}\right)\left(60\ \frac{sec}{min} * 60\ \frac{min}{hr} * 24\ \frac{hr}{day}\right)$

$\cong 0.864\ m/day$ [illegible]

$q = 0.864\ m/day * 10\ m * \left(\frac{5}{12}\right) * 1\ \frac{m}{m} = 3.6\ m^3/(day \cdot m)$



Part (c); To find the pressure head at point D (Fig.), We refer to the flow net shown in Fig. of flow net, the pressure head is equal to (10 + 3.34 m) minus the hydraulic head loss. Point D Concides with the third equipotential line begining with the upstream side. which means that the hydraulic head loss at the point is :

$$2\left(\frac{h}{N_d}\right) = 2 * \left(\frac{10}{12}\right) = 1.67 \text{ m} \quad \text{so;}$$

عدد المربعات (drops) للوصول الى نقطة D

pressure head at D = 13.34 − 1.67 = 11.67 m

similarly ;

pressur head at $E = (10 + 3.34) - 3\left(\frac{10}{12}\right) = 10.84$ m

at $F = (10 + 1.67) - 3.5\left(\frac{10}{12}\right) = 8.75$ m

drops عدد

(Note that the point F is approximately midway the forth and fifth equipotential lines starting from the upstream side).

at $G = (10 + 1.67) - 8.5\left(\frac{10}{12}\right) = 4.56$ m

at $H = (10 + 3.34) - 9\left(\frac{10}{12}\right) = 5.84$ m

at $I = (10 + 3.34) - 10\left(\frac{10}{12}\right) = 5$ m

unit length of dam

$U = \gamma_w$ (area of the pressure head diagram) (1)

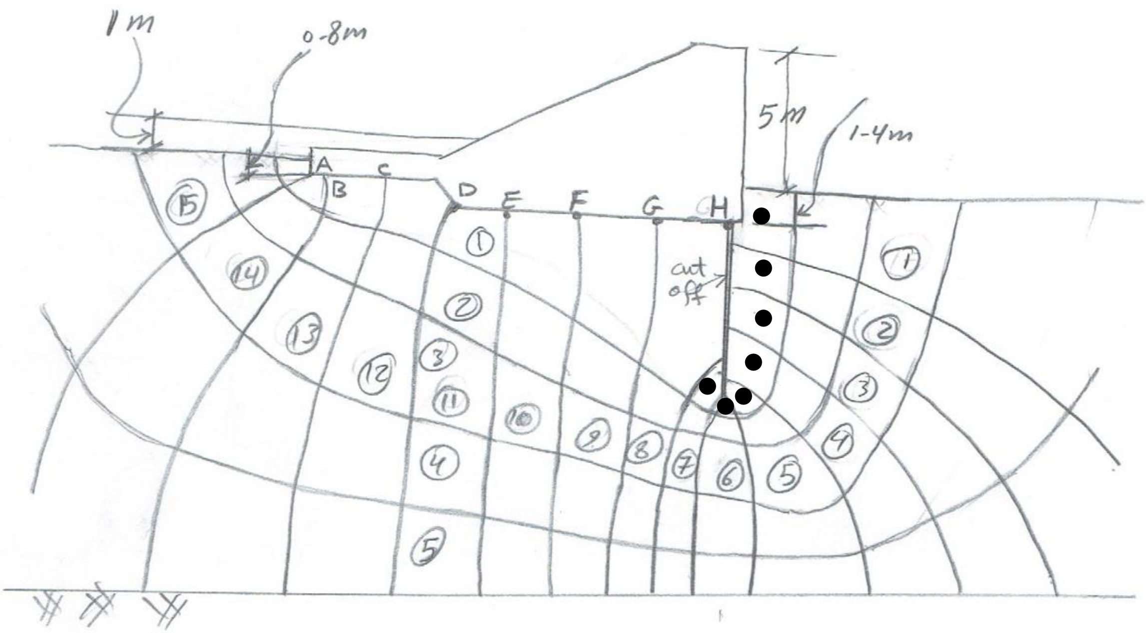
$$U = 9.81\left[\left(\frac{11.67+10.84}{2}\right)(1.67) + \left(\frac{10.84+8.75}{2}\right)(1.67) + \left(\frac{8.75+4.56}{2}\right)(18.32) + \left(\frac{4.56+5.84}{2}\right)(1.67) + \left(\frac{5.84+5}{2}\right)(1.67)\right]$$

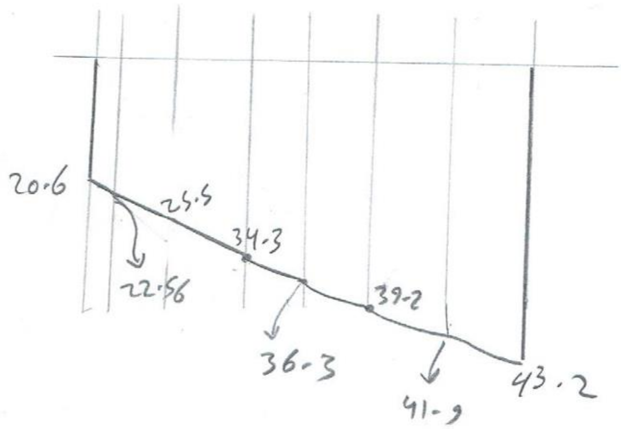
$$U = 9.81\,(18.8 + 16.36 + 121.92 + 8.68 + 9.05)$$

$$= \underline{1714.9 \text{ kN}}$$

Ex:- The section through a dam shown in Fig..
Determine the quantity of seepage under the dam and plot the distribution of uplift pressure on the base of the dam. The Coefficient of permeability of the foundation soil is $2.5 * 10^{-5}$ m/s.

Sol.





Not to scale

Sol: $N_f = 5$; $N_d = 15$; $h = 5 - 1 = 4m$

$$\therefore q = kh\frac{N_f}{N_d} = 2.5 * 10^{-5} * 4 * \frac{5}{15}$$

$$= 3.333 * 10^{-5} \; m^3/s \text{ (Per m)}.$$

The pore water pressure is calculated at the points of intersection of the equipotentials with the base of the dam.

تم اختيار نقاط تقاطع ال equipotential lines مع السد لحساب ضغط الماء على السد.

$\Delta h = 5 - 1 = 4m$

Point	h = عمود الماء فوق النقطة	Z = الخسارة بالطاقة لحين الوصول الى النقطة	h − Z	$u = \gamma_w (h - z)$
A	5.8	$z = 14 * \frac{4}{15} = 3.7$	2.1	20.6
B	5.8	$z = 13 * \frac{4}{15} = 3.5$	2.3	22.56
C	5.8	$z = 12 * \frac{4}{15} = 3.2$	2.6	25.5
D	6.4	$z = 11 * \frac{4}{15} = 2.9$	3.5	34.3
E	6.4	$z = 10 * \frac{4}{15} = 2.7$	3.7	36.3
F	6.4	$z = 9 * \frac{4}{15} = 2.4$	4	39.2
G	6.4	$z = 8 * \frac{4}{15} = 2.13$	4.27	41.9
H	6.4	$z = 7.5 * \frac{4}{15} = 2$	4.4	43.2

ملاحظة مهمة / في اي سؤال يُطلب فيه توزيع الضغط يتم أخذ نقاط تقاطع خطوط تساوي الطاقة equipotential lines مع المنشأ. أما اذا كان المنشأ يحتوي على نقاط مميزة كما في المثال الاول الذي تم حله فيتم اخذ هذه النقاط وبصورة عامة يفضل أخذ نقاط التقاطع مع خطوط تساوي الطاقة

Unit Three
Stresses in Soil Mass

Estimation of the increase in stress at various points in a soil mass due to external loading by using the theory of elasticity is an important part in the safe design of the foundations of structures. The ideal assumption of the theory of elasticity, namely, that the medium is homogeneous, elastic, and isotropic, is not quite true for most natural soil profiles. However, it provides a close estimation for geotechnical engineers, and with the use of proper safety factors, safe designs can be developed.

1- Soil Itself (Surcharge)

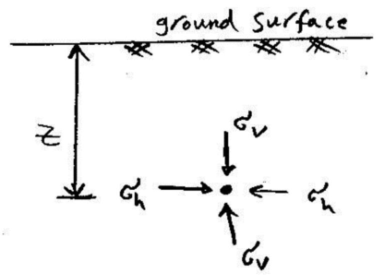
- Total vertical stress

The total vertical stress acting at a point below the ground surface is due to the weight of everything lying above: soil, water and surface loading. Total stresses are calculated from the unit weight of the soil.

A- Total stress in homogeneous soil

total stress increases with depth and with unit weight, vertical total stress at depth z,

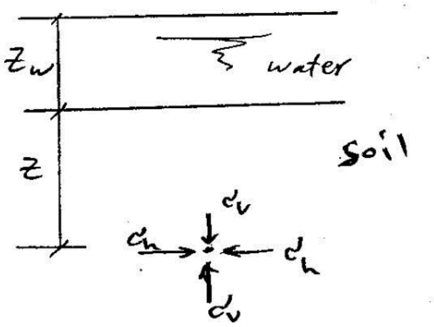
$$\sigma_v = \gamma \cdot z$$



B- Total stress below a river or lake

the total stress is the sum of the soil up to the surface and the weight of water above this:
vertical total stress at depth z,

$$\sigma_v = \gamma \cdot z + \gamma_w z_w$$



where;

γ = unit weight of the saturated soil,

γ_w = unit weight of water.

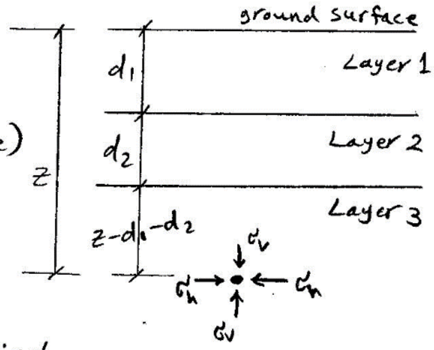
C- Total stress in multi- layered soil

$$\sigma_v = \sum \gamma \cdot \Delta z$$

$$\sigma_v = \gamma_1 d_1 + \gamma_2 d_2 + \gamma_3 (z - d_1 - d_2)$$

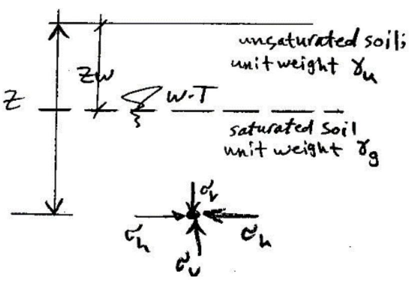
where;

$\gamma_1, \gamma_2, \gamma_3$, etc. = unit weight of soil layers 1, 2, 3, etc. respectively.



D- Total stress in unsaturated soil

$$\sigma_v = \gamma_u \cdot z_w + \gamma_g (z - z_w)$$



— Total Horizontal Stress:

The ratio of horizontal to vertical stress is expressed by a factor called the "Coefficient of lateral Stress or Lateral Stress Ratio" and is denoted by the symbol "K"

$$K = \frac{\sigma_h}{\sigma_v}$$

Three Possible cases of K values

1- No lateral strain; K_o (0.4 – 0.5) and called "Coefficient of lateral stress at rest"

2- with strain, soil compressing, K_p, its about (3) and called "active Coefficient of lateral stress"

– Pore Pressure:

The water in the pores of soil is called "Pore water" (الماء المشبع). The pressure within this pore water is called "Pore Pressure (u)" (الضغط المشبع), The magnitude of pore pressure depends on:

- the depth below the water table.
- the conditions of seepage flow.

– Effective Stresses:

When a load is applied to soil, it is carried by the water as well as the solid grains. The increase in pressure within the pore water causes drainage (flow out of the soil), and the load is transferred to the solid grains. The rate of drainage depends on the permeability of the soil. The strength and compressibility of the soil depend on the stresses within the solid granular fabric. These are called "Effective Stresses".

* The difference between the total stress and the pore pressure is called the effective stress:

effective stress = total stress – pore pressure

or $$\sigma' = \sigma - u$$

and $$u = z_w \gamma_w$$

* Changes in effective stress:

In some analyses, it is better to work in changes of quantity, rather than in absolute quantities; the effective stress expression then becomes:

$$\Delta\sigma' = \Delta\sigma - \Delta u$$

2- Point Load

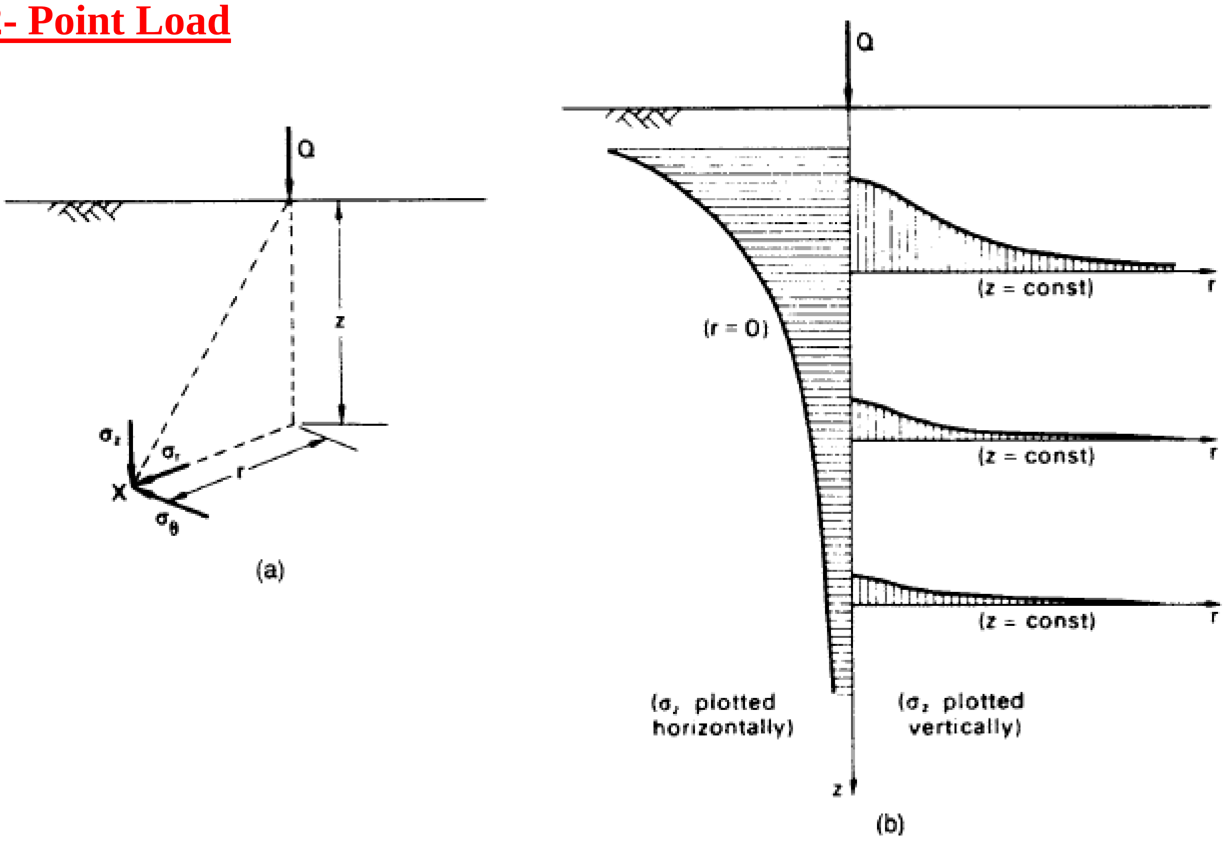


Figure 5.5 (a) Stresses due to point load and (b) variation of vertical stress due to point load.

$$I_p = \frac{3}{2\pi}\left\{\frac{1}{1+(r/z)^2}\right\}^{5/2}$$

Then,

$$\sigma_z = \frac{Q}{z^2} I_p$$

Table 5.1 Influence factors for vertical stress due to point load

r/z	I_p	r/z	I_p	r/z	I_p
0.00	0.478	0.80	0.139	1.60	0.020
0.10	0.466	0.90	0.108	1.70	0.016
0.20	0.433	1.00	0.084	1.80	0.013
0.30	0.385	1.10	0.066	1.90	0.011
0.40	0.329	1.20	0.051	2.00	0.009
0.50	0.273	1.30	0.040	2.20	0.006
0.60	0.221	1.40	0.032	2.40	0.004
0.70	0.176	1.50	0.025	2.60	0.003

3- Line Load

$$\sigma_z = \sigma_r \cos^2\theta + \sigma_\theta \sin^2\theta - 2\tau_{r\theta}\sin\theta\cos\theta = \frac{2q}{\pi r}\cos^3\theta$$

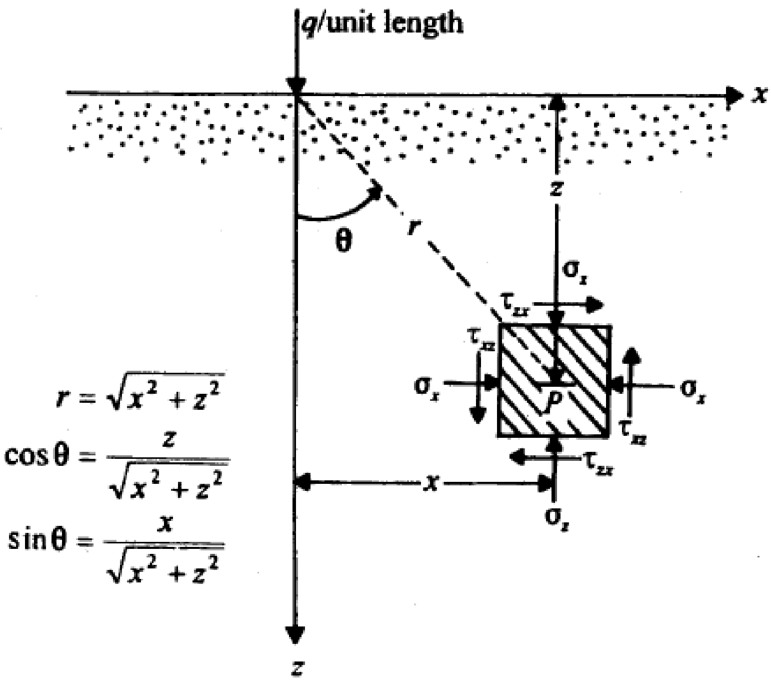
$$= \frac{2q}{\pi\sqrt{x^2+z^2}}\left(\frac{z}{\sqrt{x^2+z^2}}\right)^3 = \boxed{\frac{2qz^3}{\pi(x^2+z^2)^2}}$$

$$\sigma_x = \sigma_r \sin^2\theta + \sigma_\theta \cos^2\theta + 2\tau_{r\theta}\sin\theta\cos\theta$$

$$\sigma_x = \boxed{\frac{2qx^2z}{\pi(x^2+z^2)^2}}$$

$$\tau_{xz} = -\sigma_\theta \sin\theta\cos\theta + \sigma_r \sin\theta\cos\theta + \tau_{r\theta}(\cos^2\theta - \sin^2\theta)$$

$$\tau_{xz} = \boxed{\frac{2qxz^2}{\pi(x^2+z^2)^2}}$$



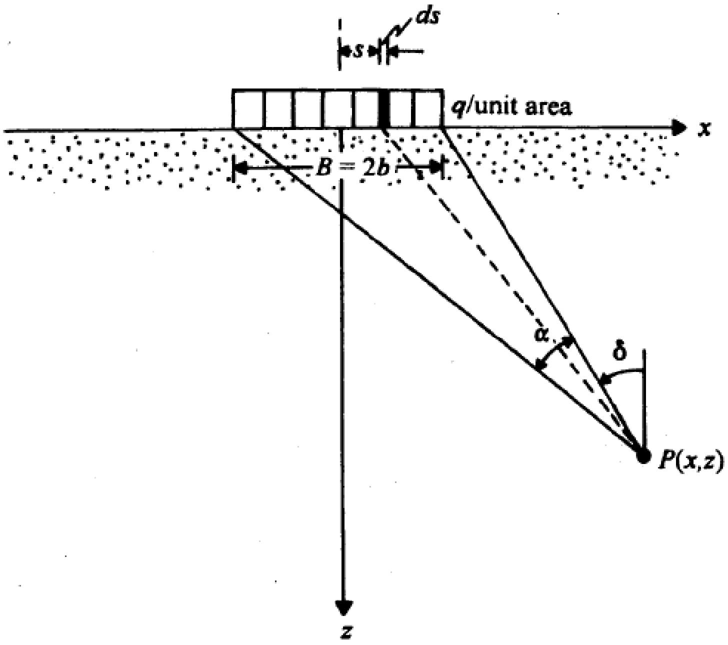
Stresses due to a vertical line load in rectangular coordinates.

4- Infinit Strip Load (continuous Footing)

$$\sigma_z = \frac{q}{\pi}[\alpha + \sin\alpha\cos(\alpha + 2\delta)]$$

$$\sigma_x = \frac{q}{\pi}[\alpha - \sin\alpha\cos(\alpha + 2\delta)]$$

$$\tau_{xz} = \frac{q}{\pi}[\sin\alpha\sin(\alpha + 2\delta)]$$



Uniform vertical loading on an infinite strip.

5- Rectangular Area Carrying Uniform Pressure

A solution has been obtained for the vertical stress at depth z under a *corner* of a rectangular area of dimensions mz and nz (Figure 5.10) carrying a uniform pressure q. The solution can be written in the form

$$\sigma_z = qI_r$$

Values of the influence factor I_r in terms of m and n are given in the chart, due to Fadum [5], shown in Figure 5.10. The factors m and n are interchangeable. The chart can also be used for a strip area, considered as a rectangular area of infinite length. Superposition enables any area based on rectangles to be dealt with and enables the vertical stress under any point within or outside the area to be obtained.

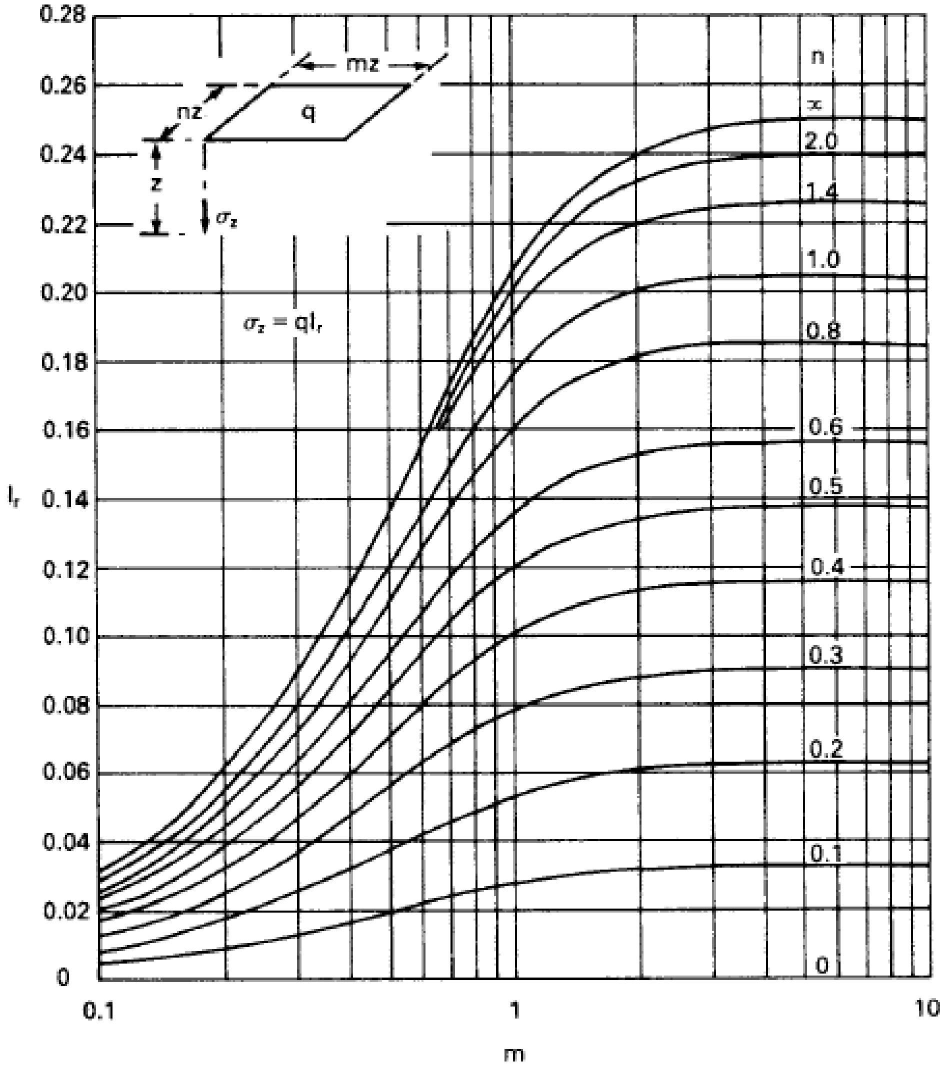


Figure 5.10 Vertical stress under a corner of a rectangular area carrying a uniform pressure. (Reproduced from R.E. Fadum (1948) *Proceedings of the 2nd International Conference of SMFE*, Rotterdam, Vol. 3, by permission of Professor Fadum.)

6- Center of a Uniformly Circular Loaded Area

Referring to Fig. (6-13) :-

$$dP = \frac{3(q\,r\,dr\,d\alpha)}{2\pi} \; \frac{z^3}{(r^2+z^2)^{5/2}}$$

by integrating the above equation

So:-

$$\Delta P = q\left\{1 - \frac{1}{[(R/z)^2+1]^{3/2}}\right\}$$

The variation of $\Delta P/q$ with z/R as obtained from this eq. is given in table (6.3).

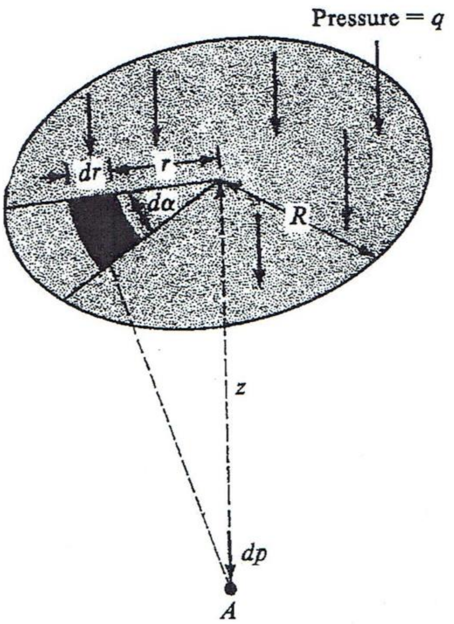


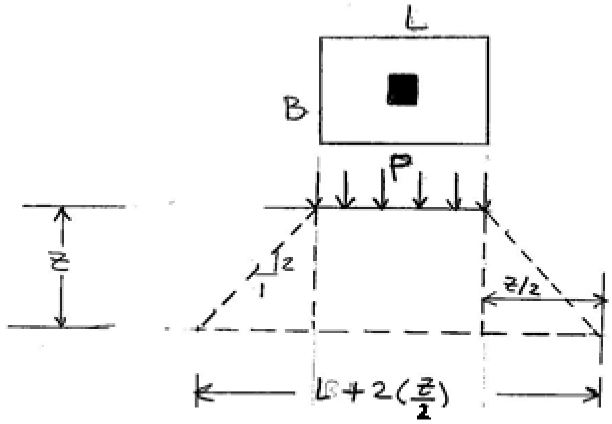
Figure 6.13 Vertical stress below the center of a uniformly loaded flexible circular area

Table 6.3 Variation of $\Delta p/q$ with z/R [Eq. (6.20)] circular

z/R	$\Delta p/q$
0	1
0.02	0.9999
0.05	0.9998
0.10	0.9990
0.2	0.9925
0.4	0.9488
0.5	0.9106
0.8	0.7562
1.0	0.6465
1.5	0.4240
2.0	0.2845
2.5	0.1996
3.0	0.1436
4.0	0.0869
5.0	0.0571

7- Slope 2:1 Method to Calculate Δp under L×B Area

$$\Delta P = \frac{P}{(B+z)(L+z)}$$



where

P = total load applied to foundation member

B, L = Footing dimensions

z = depth from footing base to elevation in soil

Example 1:

A load of 1500 kN is carried on a foundation 2 m square at a shallow depth in a soil mass. Determine the vertical stress at a point 5 m below the centre of the foundation (a) assuming that the load is uniformly distributed over the foundation and (b) assuming that the load acts as a point load at the centre of the foundation.

(a) Uniform pressure,

$$q = \frac{1500}{2^2} = 375\,\text{kN/m}^2$$

The area must be considered as four quarters to enable Figure 5.10 to be used. In this case

$$mz = nz = 1\,\text{m}$$

Then, for $z = 5\,\text{m}$

$$m = n = 0.2$$

From Figure 5.10,

$$I_r = 0.018$$

Hence,

$$\sigma_z = 4qI_r = 4 \times 375 \times 0.018 = 27\,\text{kN/m}^2$$

(b) From Table 5.1, $I_p = 0.478$ since $r/z = 0$ vertically below a point load.

Hence,

$$\sigma_z = \frac{Q}{z^2} I_p = \frac{1500}{5^2} \times 0.478 = 29\,\text{kN/m}^2$$

Example 2:

A rectangular foundation 6×3 m carries a uniform pressure of 300 kN/m² near the surface of a soil mass. Determine the vertical stress at a depth of 3 m below a point (A) on the centre line 1.5 m outside a long edge of the foundation (a) using influence factors

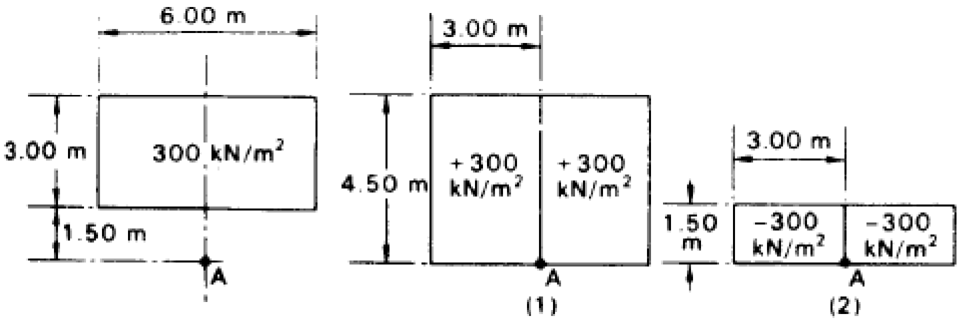


Figure 5.12 Example 5.2.

(a) Using the principle of superposition the problem is dealt with in the manner shown in Figure 5.12. For the two rectangles (1) carrying a *positive* pressure of 300 kN/m^2, $m = 1.00$ and $n = 1.50$, and therefore

$$I_r = 0.193$$

For the two rectangles (2) carrying a *negative* pressure of 300 kN/m^2, $m = 1.00$ and $n = 0.50$, and therefore

$$I_r = 0.120$$

Hence,

$$\sigma_z = (2 \times 300 \times 0.193) - (2 \times 300 \times 0.120)$$
$$= 44\,\mathrm{kN/m^2}$$

Example 3

Ex

Fig. below shows two line load on the ground surface. Determine the increase of stress at point A

Solution

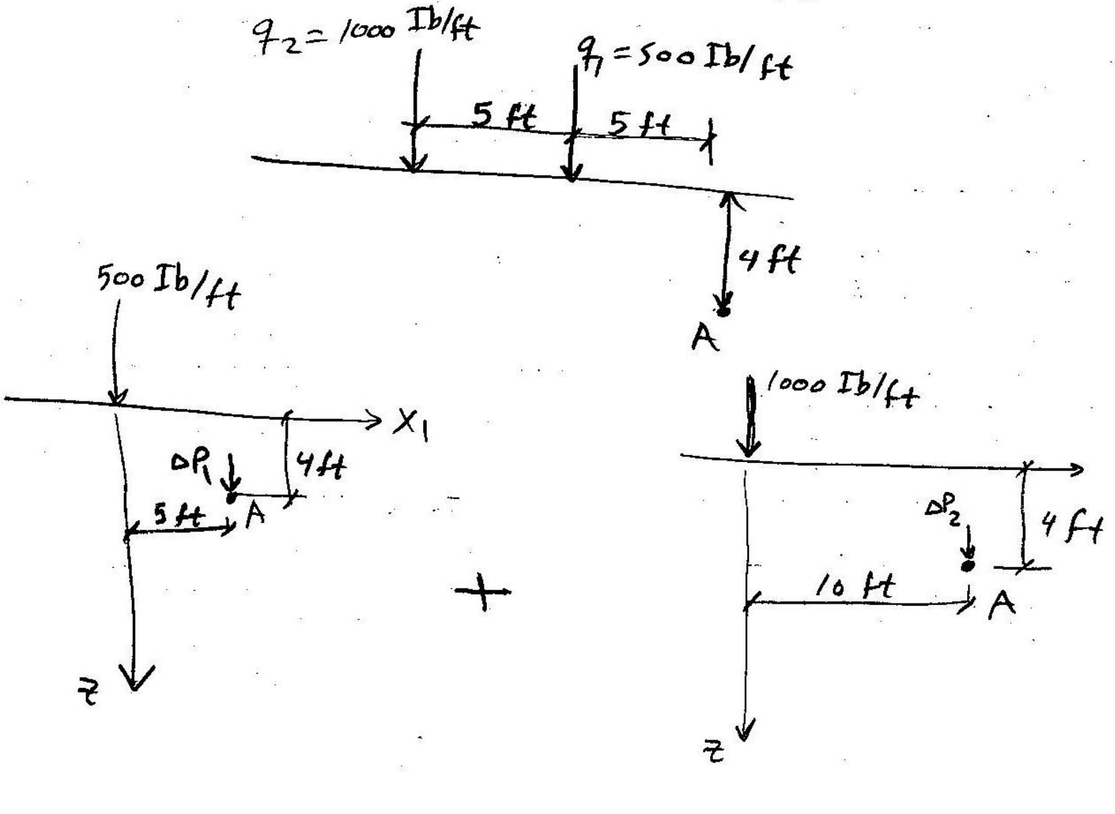
Refer to Fig. The total stress at A is:

$$\Delta P = \Delta P_1 + \Delta P_2$$

$$\Delta P_1 = \frac{2 q_1 z^3}{\pi (x_1^2 + z^2)^2} = \frac{2(500)(4)^3}{\pi (5^2 + 4^2)^2} = 12.12 \ \text{Ib/ft}^2$$

$$\Delta P_2 = \frac{2 q_2 z^3}{\pi (x_2^2 + z^2)^2} = \frac{2(1000)(4)^3}{\pi (10^2 + 4^2)^2} = 3.03 \ \text{Ib/ft}^2$$

$$\therefore \Delta P = 12.12 + 3.03 = 15.15 \ \text{Ib/ft}^2$$



Example 4

with reference to Figure 6.10 ; given ;

$q = 96$ kN/m², $B = 4$ m, and $z = 2$ m.

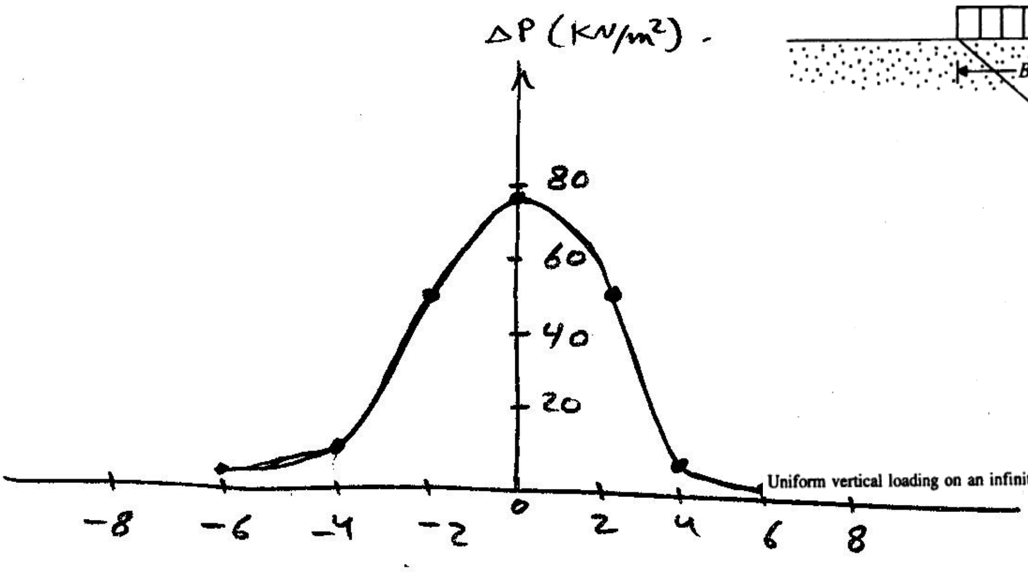
Determine the stress increase at

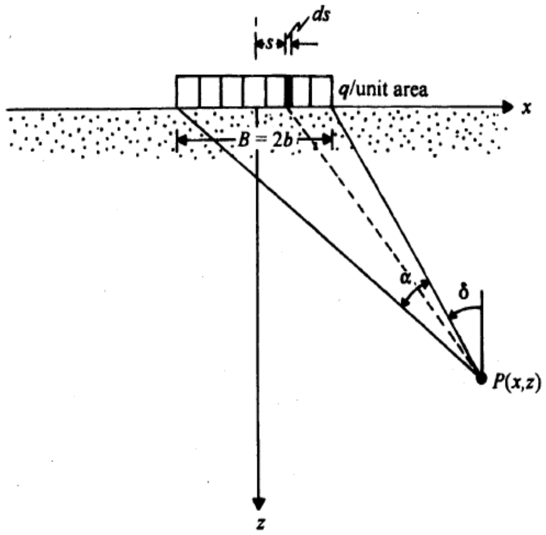
$x = \pm 6$ m ; ± 4 m ; ± 2 m , and 0 m . plot the graph ΔP against x.

From table (6-2)

X (m)	$\frac{2X}{B}$	$\frac{2Z}{B}$	$\frac{\Delta P}{q}$	ΔP (kN/m²)
± 6	± 3	1	0.0171	1.64
± 4	± 2	1	0.0776	7.45
± 2	± 1	1	0.4797	46.05
0	0	1	0.8183	78.56

The plot of ΔP vs. x is given in Figure below :-





Uniform vertical loading on an infinite strip.

Plot of ΔP against distance , x .

Unit Four
Consolidation Theory

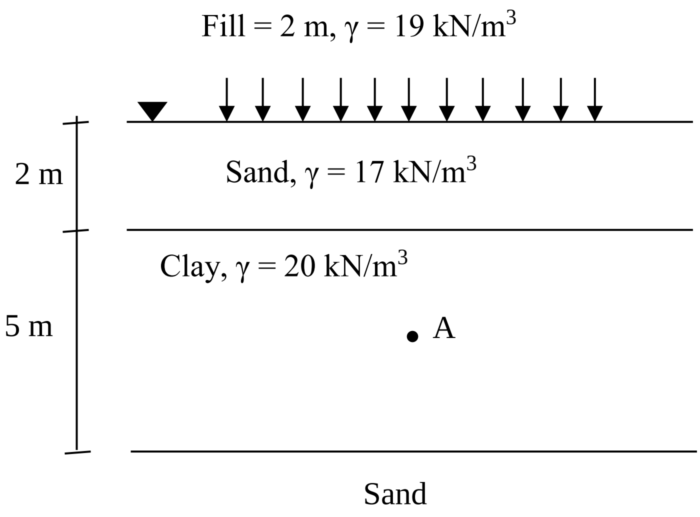
Introduction

consolidation is the gradual reduction in volume of a fully saturated soil of low permeability due to drainage of some of the pore water, the process continuing until the excess pore water pressure set up by an increase in total stress has completely dissipated; the simplest case is that of one-dimensional consolidation, in which a condition of zero lateral strain is implicit. The process of swelling, the reverse of consolidation, is the gradual increase in volume of a soil under negative excess pore water pressure. Consolidation settlement is the vertical displacement of the surface corresponding to the volume change at any stage of the consolidation process. Consolidation settlement will result, for example, if a structure is built over a layer of saturated clay or if the water table is lowered permanently in a stratum overlying a clay layer.

تعاني طبقات التربة المعرضة الى اجهادات تحميل من انضغاطية معينة. هذه الانضغاطية قد تكون ناتجة عن تشوه في حبيبات التربة، إعادة ترتيب الحبيبات، أو خروج الماء أو الهواء من الفراغات.

الانضمام: هو النقصان التدريجي في حجم تربة مشبعة واطئة النفاذية بسبب نضوح (تسرب) ضغط الماء الزائد من التربة. أبسط حالة من حالات الانضمام هي الانضمام باتجاه واحد، حيث يحدث النضوح باتجاه عمودي.

Ex 1. For the profile below: calculate the stresses at point A before and after applying the load at different times ($t = 0$, $\infty > t > 0$, $t = \infty$)



1) before applying fill

$\sigma = 2 \times 17 + 3 \times 20 = 94$ kN/m^2

$u = 5 \times 10 = 50$ kN/m^2

$\sigma' = P_o = 94 - 50 = 44$ kN/m^2

2) Immediately after applying fill ($t = 0$),

Fill loading $= 2 \times 19 = 38$ kN/m^2

$\sigma = 94 + 38 = 132$ kN/m^2

$u = 50 + 38 = 88$ kN/m^2

$\sigma' = P'_o = 132 - 88 = 44$ kN/m^2

ملاحظات:

1- الزيادة في الاجهاد الكلي $\Delta\sigma$ انتقلت مباشرة الى ضغط ماء المسام مسببة زيادته بقيمة تساوي قيمة الزيادة في الاجهاد الكلي عند $t = 0$.

2- الزيادة في الاجهاد الكلي $\Delta\sigma$ تساوي الزيادة في شدة التحميل السطحي وذلك لكون الحمل السطحي هو حمل منتشر (Fill) مسلط على سطح التربة.

3- الزيادة الأولية في ضغط ماء المسام initial excess pore water pressure ناتجة عن النفاذية القليلة للتربة الطينية المشبعة و يرمز لهذه الزيادة u_i. أي ان

$\Delta q_{s\ net}$ fill $= \Delta\sigma = u_i$

4- الاجهاد الفعال σ' لم يتأثر عند زمن $t = 0$ بالزيادة الحاصلة بالتحميل السطحي و ذلك لأن كل الزيادة قد انتقلت الى ضغط ماء المسام مسببة زيادته.

5- نسبة الفجوات (e) لم تتأثر لعدم تأثر الاجهاد الفعال σ' و يرمز لها بـ e_o .

3) For a specific time after applying load (Fill), ($\infty > t > 0$);

$\sigma = 94 + 38 = 132$ kN/m^2

$u = 50 + x$

$\sigma' = P'_o = 44 + (38 - x)$

ملاحظات:

1- الزيادة في الاجهاد الكلي $\Delta\sigma$ لا تزال موجودة لعدم تغير الحمل السطحي.

2- الزيادة في ضغط ماء المسام بدأت تقل تدريجيا و ذلك لنضوح الضغط الزائد في الماء الى المناطق ذات الضغط الأقل (الرمل ذو النفاذية الأعلى).

3- الزيادة المتبقية في ضغط ماء المسام بعد زمن معين ($\infty > t > 0$) تدعى excess pore water pressure ويرمز لها u_e.

4- بدأت تظهر زيادة تدريجية في الاجهاد الفعال ($\Delta P' = \Delta\sigma'$ at time t).

5- مجموع الزيادة المتبقية في ضغط ماء المسام Ue والزيادة الحاصلة في الاجهاد الفعال بعد زمن (t) يجب أن تساوي الزيادة في الاجهاد الكلي $\Delta\sigma$. أي ان $\Delta\sigma = u_e + \Delta P'$ at any time.

6- يرافق الزيادة في الجهاد الفعال نقصان تدريجي في نسبة الفجوات (e).

4) After a very long time from applying fill ($t = \infty$);

$\sigma = 94 + 38 = 132$ kN/m^2

$u = 50$

$\sigma' = P'_f = 44 + 38 = 82$ kN/m^2

ملاحظات:

1- الزيادة في الاجهاد الكلي $\Delta\sigma$ لا تزال موجودة لعدم تغير الحمل السطحي.

2- الزيادة في ضغط ماء المسام أصبحت تساوي (صفر) أي أن كل ضغط الماء الزائد (عن حالة الاستقرار) قد تسرب الى المناطق ذات الضغط الأقل (الرمل ذو النفاذية الأعلى).

3- كل الزيادة في الاجهاد الكلي $\Delta\sigma$ قد انتقلت الى الاجهاد الفعال ويشار اليها بالرمز $\Delta\sigma, p'$.

4- الاجهاد الفعال النهائي أصبح $P'_f = P'_o + \Delta P'$

5- الزيادة في الاجهاد الفعال بعد زمن بعيد $t = \infty$ $(\Delta P')$ هي نفسها الزيادة في الاجهاد الكلي $(\Delta\sigma)$ وهي نفسها الزيادة الأولية في ضغط ماء المسام u_i بمعنى آخر

$$\Delta\sigma = \Delta P'_f = u_i$$

6- يتسرب جميع ضغط الماء الزائد وبالتالي انتقال كل $\Delta\sigma$ الى الاجهاد الفعال تنتهي عملية الانضمام وهذا يستغرق فترة طويلة قد تصل الى عدة سنوات وتكون عندها درجة الانضمام Degree of Consolidation تساوي 100% ويمكن تعريف درجة الانضمام كالتالي:

$$u_z\ \% = \left(\frac{u_i - u_e}{u_i}\right) \times 100\% \qquad or \qquad u_z\ \% = \left(1 - \frac{u_e}{u_i}\right) \times 100\%$$

u_i : Degree of consolidation
u_e : Excess pore water pressure at time (t)
u_i : initial excess pore water pressure at time (t =0)

7- نسبة الفجوات النهائية e_f أصبحت تساوي $e_f = e_o - \Delta e$

حيث أن Δe تمثل التغير في نسبة الفجوات بين حالة قبل وضع الحمل ونسبة الفجوات بعد انتهاء عملية الانضمام.

The Oedometer Test

The characteristics of a soil during one-dimensional consolidation or swelling can be determined by means of the oedometer test. Figure 7.1 shows diagrammatically a cross-section through an oedometer. The test specimen is in the form of a disc, held inside a metal ring and lying between two porous stones. The upper porous stone, which can move inside the ring with a small clearance, is fixed below a metal loading cap through which pressure can be applied to the specimen. The whole assembly sits in an open cell of water to which the pore water in the specimen has free access. The ring confining the specimen may be either fixed (clamped to the body of the cell) or floating (being free to move vertically): the inside of the ring should have a smooth polished surface to reduce side friction. The confining ring imposes a condition of zero lateral strain on the specimen, the ratio of lateral to vertical effective stress being K_o, the coefficient of earth pressure at-rest. The compression of the specimen under pressure is measured by means of a dial gauge operating on the loading cap.

The test procedure has been standardized in BS 1377 (Part 5) [4] which specifies that the oedometer shall be of the fixed ring type. The initial pressure will depend on the type of soil, then a sequence of pressures is applied to the specimen, each being double the previous value. Each pressure is normally maintained for a period of 24 h (in exceptional cases a period of 48 h may be required), compression readings being observed at suitable intervals during this period. At the end of the increment period,

when the excess pore water pressure has completely dissipated, the applied pressure equals the effective vertical stress in the specimen. The results are presented by plotting the thickness (or percentage change in thickness) of the specimen or the void ratio at the end of each increment period against the corresponding effective stress. The effective stress may be plotted to either a natural or a logarithmic scale. If desired, the expansion of the specimen can be measured under successive decreases in applied pressure. However, even if the swelling characteristics of the soil are not required, the expansion of the specimen due to the removal of the final pressure should be measured. The void ratio at the end of each increment period can be calculated from the dial gauge readings and either the water content or the dry weight of the specimen at the end of the test. Referring to the phase diagram in Figure 7.2, the two methods of calculation are as follows.

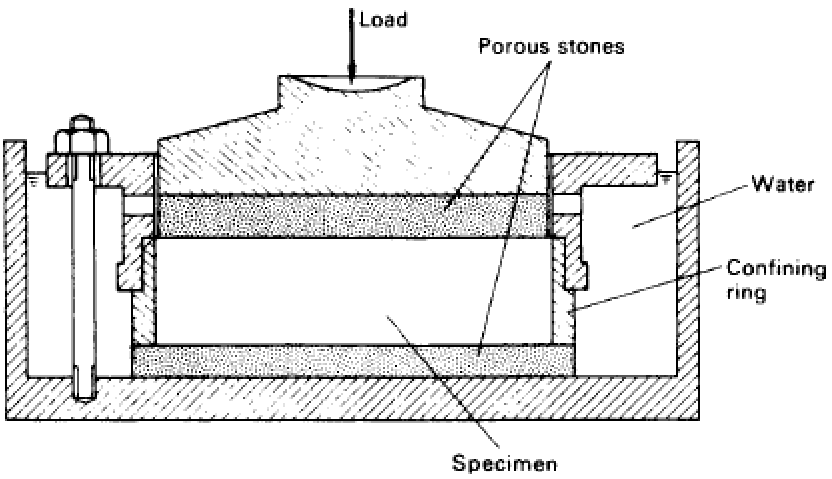


Figure 7.1 The oedometer.

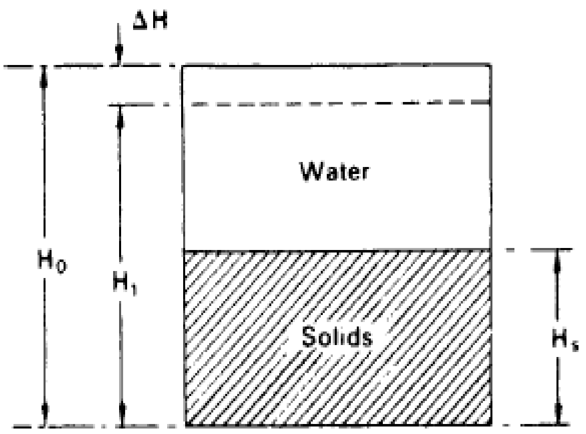


Figure 7.2 Phase diagram.

1 Water content measured at end of test $= w_1$
Void ratio at end of test $= e_1 = w_1 G_s$ (assuming $S_r = 100\%$)
Thickness of specimen at start of test $= H_0$
Change in thickness during test $= \Delta H$
Void ratio at start of test $= e_0 = e_1 + \Delta e$
where

$$\frac{\Delta e}{\Delta H} = \frac{1 + e_0}{H_0} \tag{7.1}$$

In the same way Δe can be calculated up to the end of any increment period.

2 Dry weight measured at end of test $= M_s$ (i.e. mass of solids)
Thickness at end of any increment period $= H_1$
Area of specimen $= A$
Equivalent thickness of solids $= H_s = M_s / A G_s \rho_w$
Void ratio,

$$e_1 = \frac{H_1 - H_s}{H_s} = \frac{H_1}{H_s} - 1 \tag{7.2}$$

Compressibility Charachtarestics

Typical plots of void ratio (e) after consolidation against effective stress (σ') for a saturated clay are shown in Figure 7.3, the plots showing an initial compression followed by expansion and recompression (cf. Figure 4.9 for isotropic consolidation). The shapes of the curves are related to the stress history of the clay. The e–$\log \sigma'$ relationship for a normally consolidated clay is linear (or nearly so) and is called the virgin compression line. If a clay is overconsolidated, its state will be represented by a point on the expansion or recompression parts of the e–$\log \sigma'$ plot. The recompression curve ultimately joins the virgin compression line: further compression then occurs along the virgin line. During compression, changes in soil structure continuously take place and the clay does not revert to the original structure during expansion. The plots show that a clay in the overconsolidated state will be much less compressible than that in a normally consolidated state.

The compressibility of the clay can be represented by one of the following coefficients.

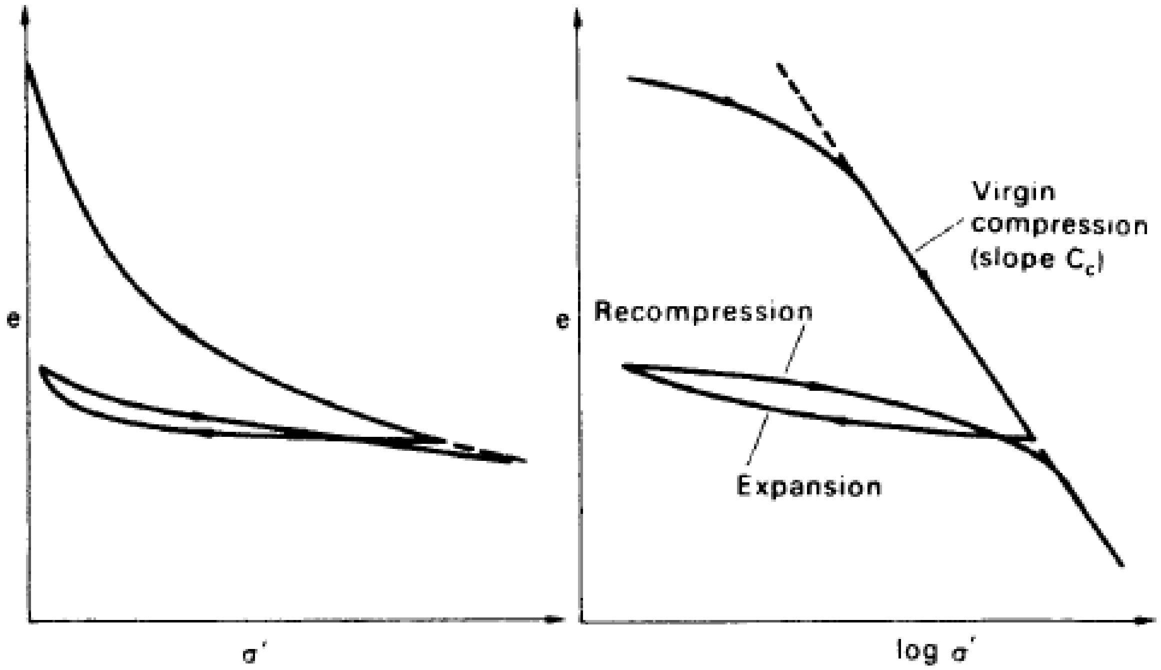


Figure 7.3 Void ratio–effective stress relationship.

1 *The coefficient of volume compressibility* (m_v), defined as the volume change per unit volume per unit increase in effective stress. The units of m_v are the inverse of pressure (m^2/MN). The volume change may be expressed in terms of either void ratio or specimen thickness. If, for an increase in effective stress from σ'_0 to σ'_1, the void ratio decreases from e_0 to e_1, then

$$m_v = \frac{1}{1+e_0}\left(\frac{e_0 - e_1}{\sigma'_1 - \sigma'_0}\right) \tag{7.3}$$

$$m_v = \frac{1}{H_0}\left(\frac{H_0 - H_1}{\sigma'_1 - \sigma'_0}\right) \tag{7.4}$$

The value of m_v for a particular soil is not constant but depends on the stress range over which it is calculated. BS 1377 specifies the use of the coefficient m_v calculated for a stress increment of 100 kN/m^2 in excess of the effective overburden pressure of the *in-situ* soil at the depth of interest, although the coefficient may also be calculated, if required, for any other stress range.

2 *The compression index* (C_c) is the slope of the linear portion of the e–log σ' plot and is dimensionless. For any two points on the linear portion of the plot

$$C_c = \frac{e_0 - e_1}{\log(\sigma'_1/\sigma'_0)} \tag{7.5}$$

The expansion part of the e–log σ' plot can be approximated to a straight line, the slope of which is referred to as the *expansion index* C_e.

Preconsolidation Pressure

Casagrande proposed an empirical construction to obtain, from the e–$\log \sigma'$ curve for an overconsolidated clay, the maximum effective vertical stress that has acted on the clay in the past, referred to as the *preconsolidation pressure* (σ'_c). Figure 7.4 shows a typical e–$\log \sigma'$ curve for a specimen of clay, initially overconsolidated. The initial curve indicates that the clay is undergoing recompression in the oedometer, having at some stage in its history undergone expansion. Expansion of the clay *in situ* may, for example, have been due to melting of ice sheets, erosion of overburden or a rise in water table level. The construction for estimating the preconsolidation pressure consists of the following steps:

1. Produce back the straight-line part (BC) of the curve.
2. Determine the point (D) of maximum curvature on the recompression part (AB) of the curve.
3. Draw the tangent to the curve at D and bisect the angle between the tangent and the horizontal through D.
4. The vertical through the point of intersection of the bisector and CB produced gives the approximate value of the preconsolidation pressure.

Whenever possible the preconsolidation pressure for an overconsolidated clay should not be exceeded in construction. Compression will not usually be great if the effective vertical stress remains below σ'_c: only if σ'_c is exceeded will compression be large.

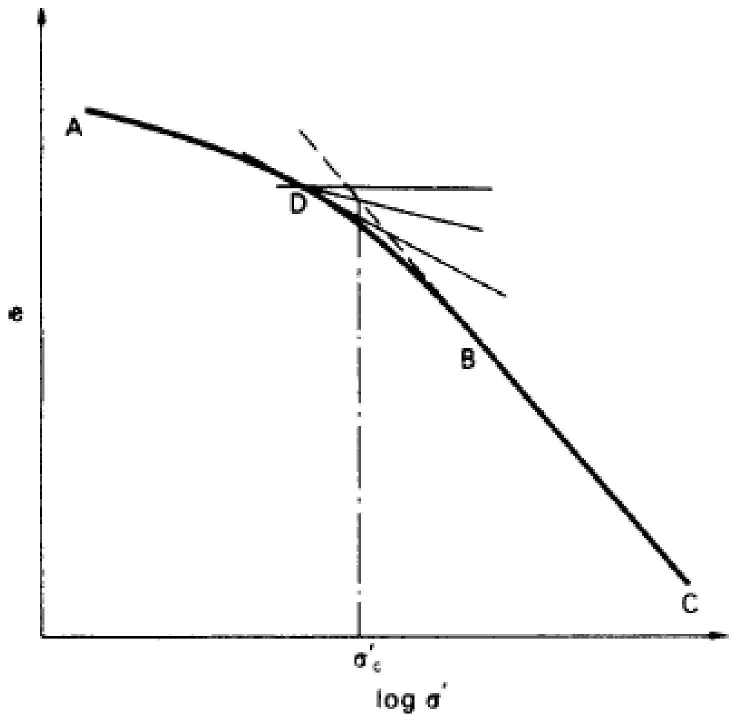


Figure 7.4 Determination of preconsolidation pressure.

Example 7.1

The following compression readings were obtained in an oedometer test on a specimen of saturated clay ($G_s = 2.73$):

Pressure (kN/m^2)	0	54	107	214	429	858	1716	3432	0
Dial gauge after 24 h (mm)	5.000	4.747	4.493	4.108	3.449	2.608	1.676	0.737	1.480

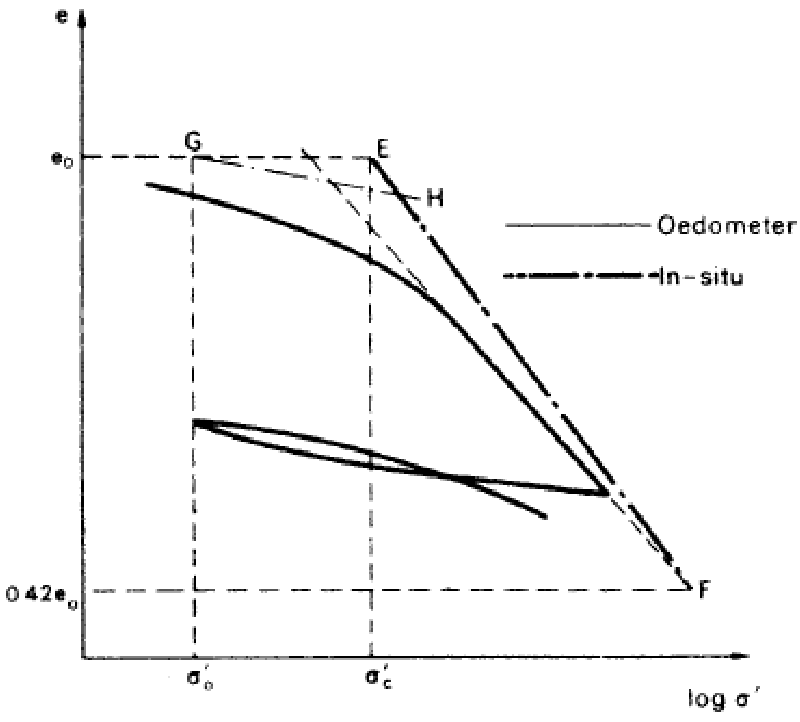


Figure 7.5 In-situ e–log σ' curve.

The initial thickness of the specimen was 19.0 mm and at the end of the test the water content was 19.8%. Plot the e–$\log\sigma'$ curve and determine the preconsolidation pressure. Determine the values of m_v for the stress increments 100–200 and 1000–1500 kN/m^2. What is the value of C_c for the latter increment?

$$\text{Void ratio at end of test} = e_1 = w_1 G_s = 0.198 \times 2.73 = 0.541$$

$$\text{Void ratio at start of test} = e_0 = e_1 + \Delta e$$

Now

$$\frac{\Delta e}{\Delta H} = \frac{1 + e_0}{H_0} = \frac{1 + e_1 + \Delta e}{H_0}$$

i.e.

$$\frac{\Delta e}{3.520} = \frac{1.541 + \Delta e}{19.0}$$

$$\Delta e = 0.350$$

$$e_0 = 0.541 + 0.350 = 0.891$$

In general, the relationship between Δe and ΔH is given by

$$\frac{\Delta e}{\Delta H} = \frac{1.891}{19.0}$$

i.e. $\Delta e = 0.0996\,\Delta H$, and can be used to obtain the void ratio at the end of each increment period (see Table 7.1). The e–$\log \sigma'$ curve using these values is shown in Figure 7.6. Using Casagrande's construction, the value of the preconsolidation pressure is 325 kN/m^2.

$$m_v = \frac{1}{1 + e_0} \cdot \frac{e_0 - e_1}{\sigma'_1 - \sigma'_0}$$

Table 7.1

Pressure (kN/m^2)	ΔH (mm)	Δe	e
0	0	0	0.891
54	0.253	0.025	0.866
107	0.507	0.050	0.841
214	0.892	0.089	0.802
429	1.551	0.154	0.737
858	2.392	0.238	0.653
1716	3.324	0.331	0.560
3432	4.263	0.424	0.467
0	3.520	0.350	0.541

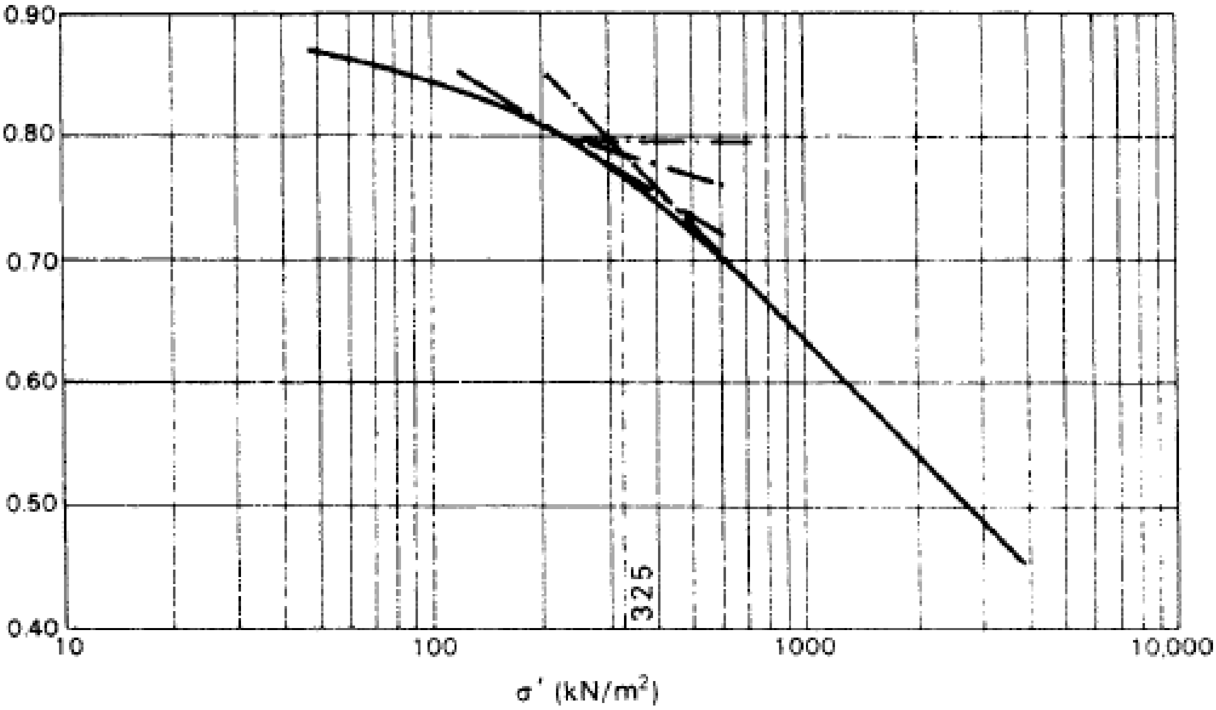


Figure 7.6 Example 7.1.

For $\sigma'_0 = 100\,\text{kN/m}^2$ and $\sigma'_1 = 200\,\text{kN/m}^2$,

$$e_0 = 0.845 \quad \text{and} \quad e_1 = 0.808$$

and therefore

$$m_v = \frac{1}{1.845} \times \frac{0.037}{100} = 2.0 \times 10^{-4}\,\text{m}^2/\text{kN} = 0.20\,\text{m}^2/\text{MN}$$

For $\sigma'_0 = 1000\,\text{kN/m}^2$ and $\sigma'_1 = 1500\,\text{kN/m}^2$,

$$e_0 = 0.632 \quad \text{and} \quad e_1 = 0.577$$

and therefore

$$m_v = \frac{1}{1.632} \times \frac{0.055}{500} = 6.7 \times 10^{-5}\,\text{m}^2/\text{kN} = 0.067\,\text{m}^2/\text{MN}$$

and

$$C_c = \frac{0.632 - 0.577}{\log(1500/1000)} = \frac{0.055}{0.176} = 0.31$$

Note that C_c will be the same for any stress range on the linear part of the e–$\log\sigma'$ curve; m_v will vary according to the stress range, even for ranges on the linear part of the curve.

Consolidation Settlement

If m_v and $\Delta\sigma'$ are assumed constant with depth, then

$$s_c = m_v \Delta\sigma' H$$

or

$$s_c = \frac{e_0 - e_1}{1 + e_0} H$$

or, in the case of a normally consolidated clay,

$$s_c = \frac{C_c \log(\sigma'_1/\sigma'_0)}{1 + e_0} H$$

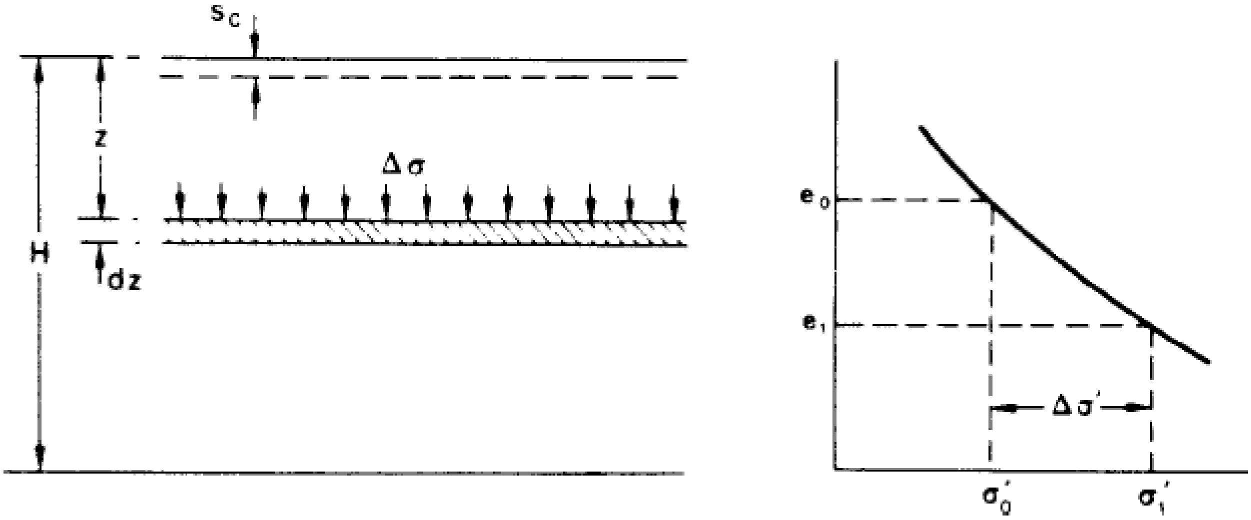


Figure 7.7 Consolidation settlement.

Degree of Consolidation

For an element of soil at a particular depth z in a clay layer the progress of the consolidation process under a particular total stress increment can be expressed in terms of void ratio as follows:

$$U_z = \frac{e_0 - e}{e_0 - e_1}$$

where U_z is defined as the degree of consolidation, at a particular instant of time, at depth z ($0 \le U_z \le 1$), and e_0 = void ratio before the start of consolidation, e_1 = void ratio at the end of consolidation and e = void ratio, at the time in question, during consolidation.

If the e–σ' curve is assumed to be linear over the stress range in question, as shown in Figure 7.15, the degree of consolidation can be expressed in terms of σ':

$$U_z = \frac{\sigma' - \sigma'_0}{\sigma'_1 - \sigma'_0}$$

$T_v = \frac{c_v t}{d^2}$ a dimensionless number called the *time factor*

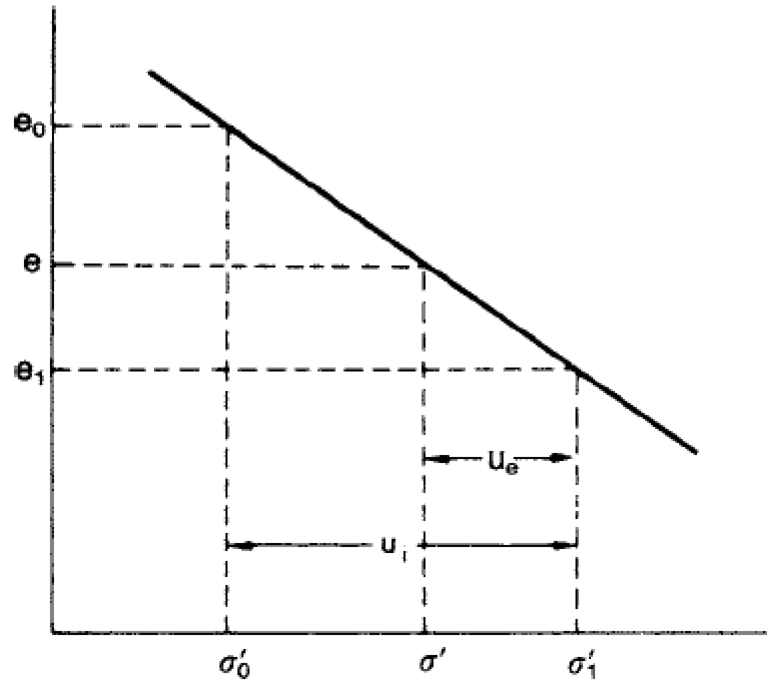


Figure 7.15 Assumed linear e–σ′ relationship.

The degree of consolidation can then be expressed as

$$U_z = \frac{u_i - u_e}{u_i} = 1 - \frac{u_e}{u_i}$$

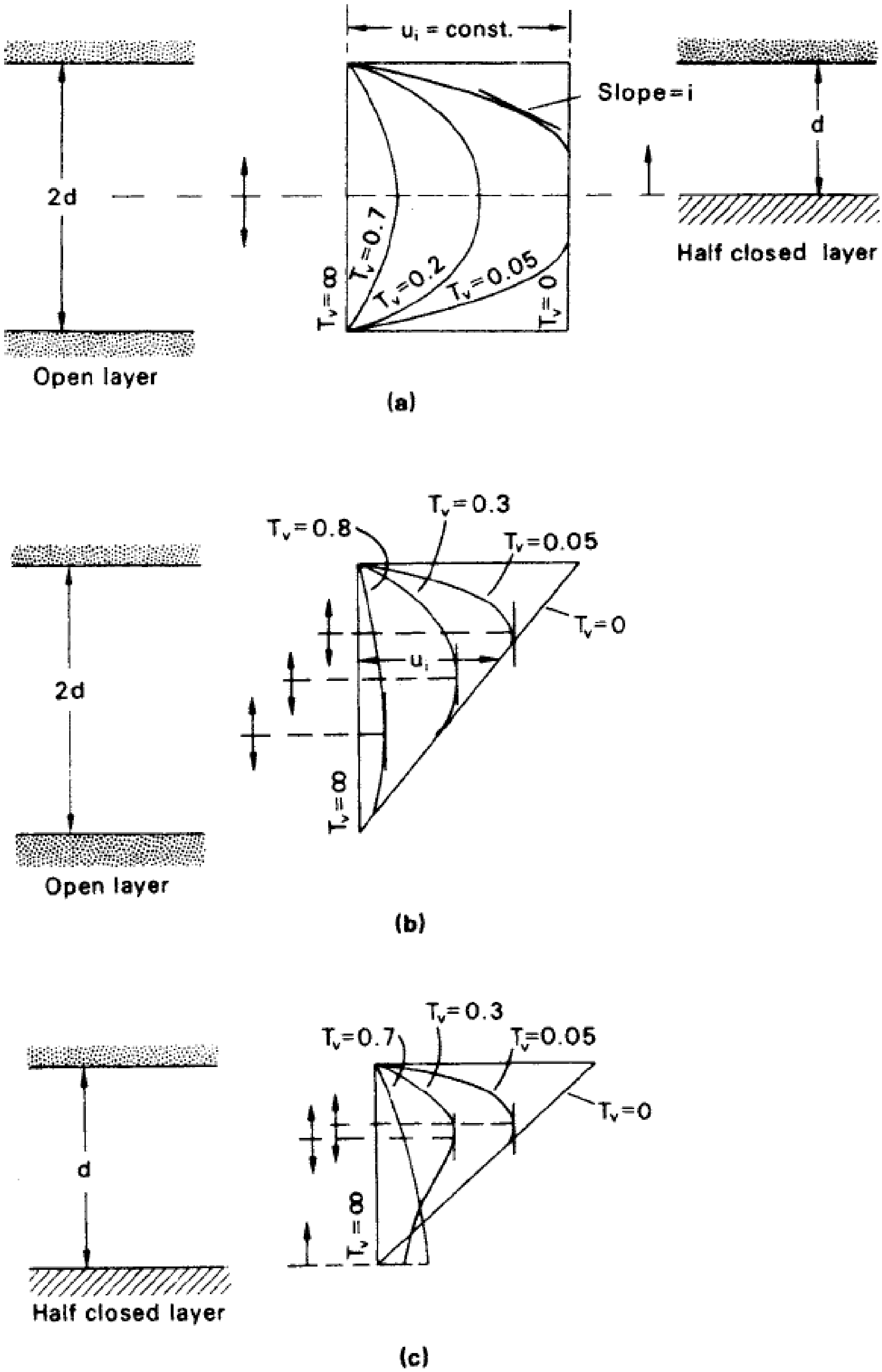


Figure 7.17 Isochrones.

$$\text{for} \quad U < 0.60, \quad T_v = \frac{\pi}{4} U^2$$

$$\text{for} \quad U > 0.60, \quad T_v = -0.933 \log(1 - U) - 0.085$$

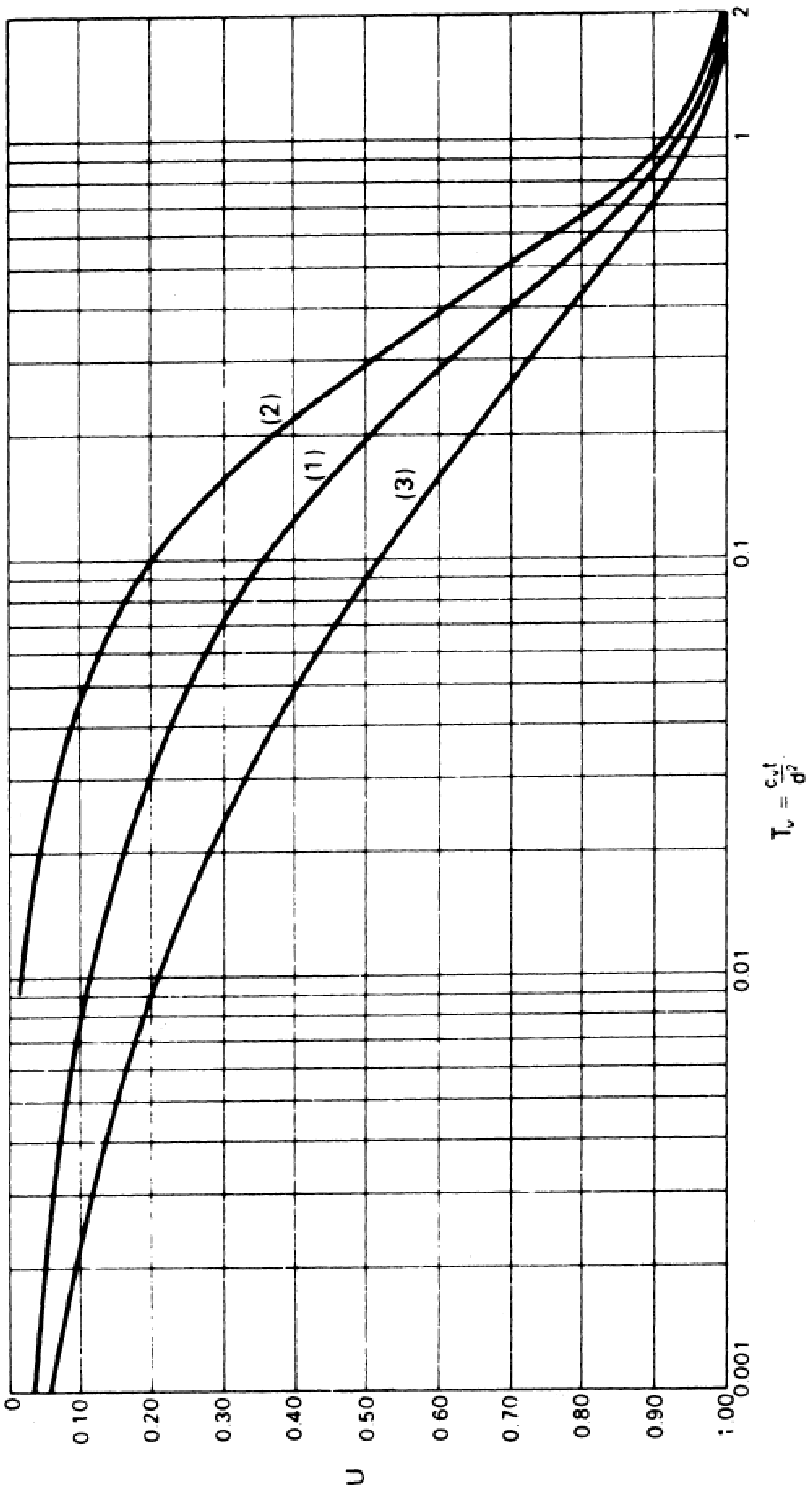


Figure 7.18 Relationships between average degree of consolidation and time factor.

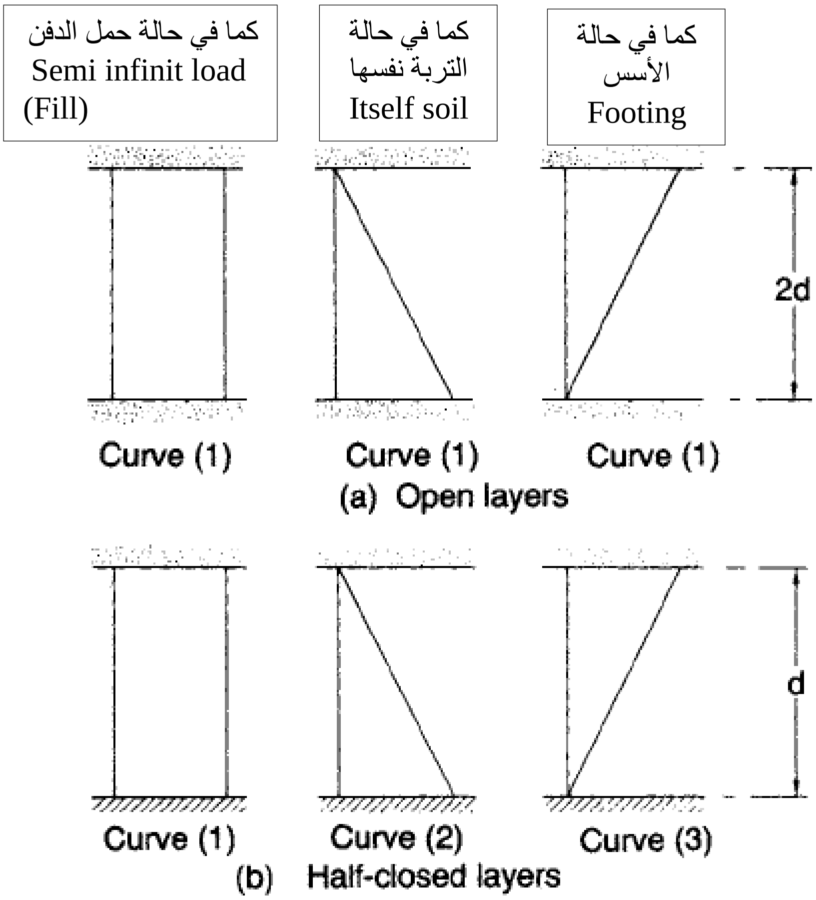
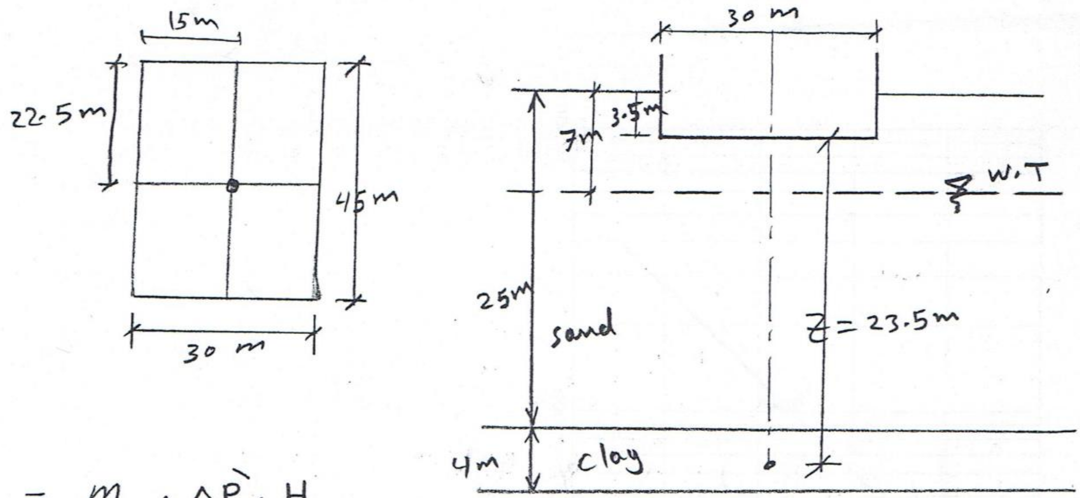


Figure 7.19 Initial variations of excess pore water pressure.

Ex

A building is supported on a raft 45 m x 30 m, the net foundation pressure being 125 kN/m². The soil profile is shown in Fig. below. The value of m_v for the clay is 0.35 m²/MN. Determine the final settlement under the center of the raft due to consolidation of the clay.



Sol.

$S = m_v \cdot \Delta P' \cdot H$

$\Delta P' = \Delta P$

$Z = 23.5\ m \quad ; \quad m = \frac{B}{Z} = \frac{15}{23.5} = 0.638 \quad ; \quad n = \frac{L}{Z} = \frac{22.5}{23.5} = 0.957$

$I_Z = 0.14$

$\therefore \Delta P = 0.14 * 125 * 4 = 70\ kN/m^2$

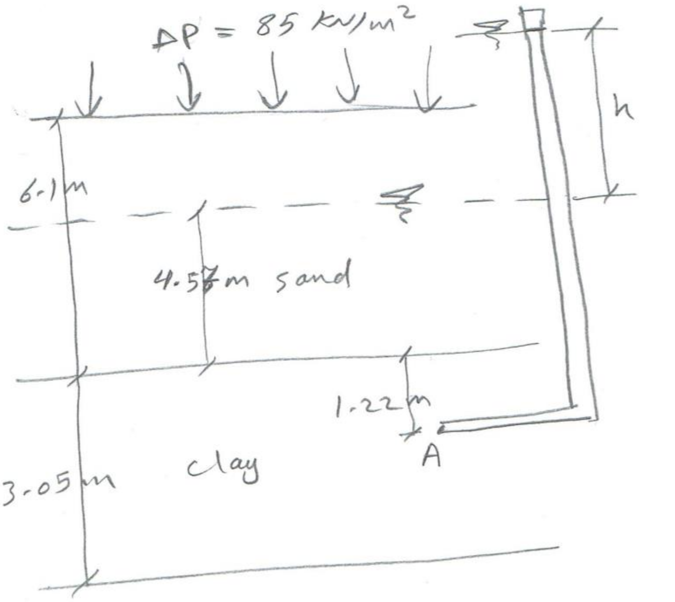
$\therefore S = m_v \cdot \Delta P \cdot H = 0.35 * 70 * 4 = 98\ mm$.

Example:- A soil profile is shown in Fig. , A surcharge load of 85 kN/m^2 is applied on the ground surface. Determin the following :

a- How high the water will rise in the peizometer immediately after the application of load.

b- what the degree of consolidation is at A when h = 6.1 m

c- find h when the degree of consolidation at A is 50 %



Solution

Part (a)

assuming uniform increase of initial excess pore water pressure through the 3.05 m of clay layer

$$u_o = \Delta\sigma = 85 \ kN/m^2$$

$$\therefore h = \frac{85}{9.81} = \underline{8.66 \ m}$$

Part (b)

$$U_A \% = \left(1 - \frac{u_A}{u_o}\right) 100$$

$$= \left(1 - \frac{6.1 * 9.81}{8.66 * 9.81}\right) 100 =$$

$$= 29.6 \%$$

Part (c)

$$U_A = 0.5 = \left(1 - \frac{U_A}{U_o}\right)$$

or

$$0.5 = \left(1 - \frac{u_A}{85}\right)$$

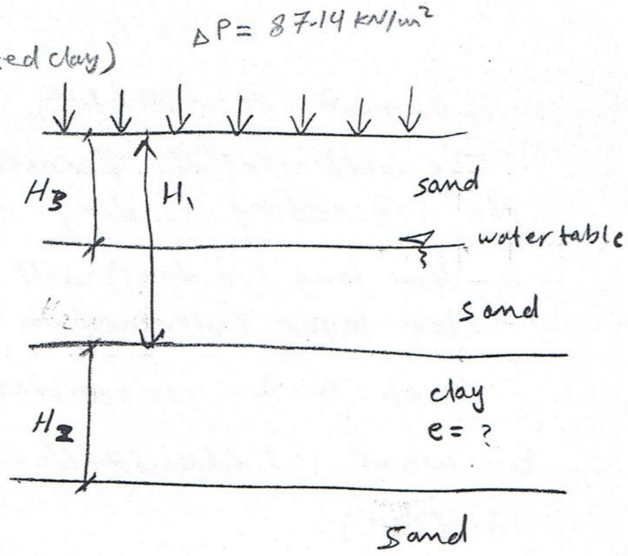
$$\therefore u_A = \underline{42.5 \ kN/m^2}$$

Hence

$$h = \frac{42.5}{9.81} = \underline{4.33 \ m}$$

Problem

Given: Assume (normally consolidated clay)



$\Delta P = 87.14 \ kN/m^2$

$H_1 = 4m$; $H_2 = 3.2m$

$H_3 = 1m$;

sand; $\gamma_{dry} = 14.6 \ kN/m^3$

$\gamma_{sat} = 17.3 \ kN/m^3$

clay: $\gamma_{sat} = 19.3 \ kN/m^3$

$LL = 38$; $e = 0.75$

what will be the settlement of the clay layer due to primary consolidation.

Sol

$$S = \frac{C_c H}{1+e_o} \log\left(\frac{P_o + \Delta P}{P_o}\right)$$

$$C_c = 0.009(LL-10) = 0.009(38-10) = 0.252$$

$$P_o = 14.6 * 1m + (17.3 - 9.81) * 3m + (19.3 - 9.81) * \frac{3.2}{2} m$$

$$= 52.254 \ kN/m^2$$

$$\therefore S = \frac{0.252 * 3.2}{1 + 0.75} \log\left(\frac{52.254 + 87.14}{52.254}\right)$$

$$= 0.196 \ m \simeq 19.6 \ cm$$

Problem 7.13 : For a normally consolidated clay given:-

$P_o = 2\ ton/ft^2 \quad e = e_o = 1.22$

$P_o + \Delta P = 4\ ton/ft^2 \quad e = 0.98$

The coefficient of permeability (K) of the clay for the Preceeding loading range is $2*10^{-4}$ ft/day.

a- How long (in days) will it take for a 10-ft thick clay layer (drained on both sides) in the field to reach 50% consolidation?

b- what is the settlement at that time (50% consolidation).

$\gamma_w = 62.4\ \frac{Ib}{ft^3} * 0.453\ \frac{kg}{Ib}$

$= 28.26\ \frac{kg}{ft^3} / 1000\ \frac{kg}{ton}$

$= 0.02826\ ton/ft^3$

Sol

$$C_v = \frac{k}{\gamma_w m_v}$$

$$m_v = \frac{a_v}{1+e_o} \Rightarrow a_v = \frac{\Delta e}{\Delta P} = \frac{1.22-0.98}{4-2} = 0.12\ \frac{ft^2}{ton}$$

$$\therefore m_v = \frac{0.12\ ft^2/ton}{1+1.22} = 0.05405\ \frac{ft^2}{ton}$$

$$\therefore C_v = \frac{2*10^{-4}\ ft/day}{0.02826 \cancel{\frac{ton}{ft^3}}_{ft} * 0.05405 \cancel{\frac{ft^2}{ton}}} = 0.1309\ ft^2/day$$

$$\therefore S = m_v \cdot \Delta P \cdot H = 0.05405\ \cancel{\frac{ft^2}{ton}} * (4-2)\ \cancel{ton/ft^2} * 10\ ft$$

$$= 1.081\ ft$$

$$\text{or } S = \frac{e_o - e_1}{1+e_o} H = \frac{1.22-0.98}{1+1.22} * 10 = 1.081\ ft$$

For $U = 0$ To 60% ; $T_v = \frac{\pi}{4}\left(\frac{U\%}{100}\right)^2 = \frac{\pi}{4}\left(\frac{50}{100}\right)^2 = 0.196$

$$T_v = \frac{C_v t}{H_{dr}^2} \Rightarrow 0.196 = \frac{0.1309\ ft^2/day * t}{\left(\frac{10}{2}\right)^2 ft^2}$$

$4.9 = 0.1309\ t \Rightarrow \quad \therefore t = 37.5$ days

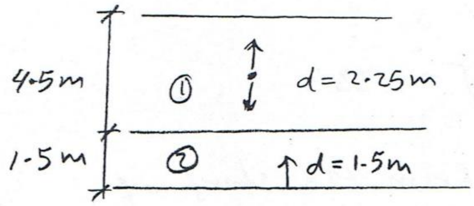
$$\therefore S_{at\ 50\%\ cons.} = U \cdot S_t = 0.5 * 1.081 = 0.54\ ft$$

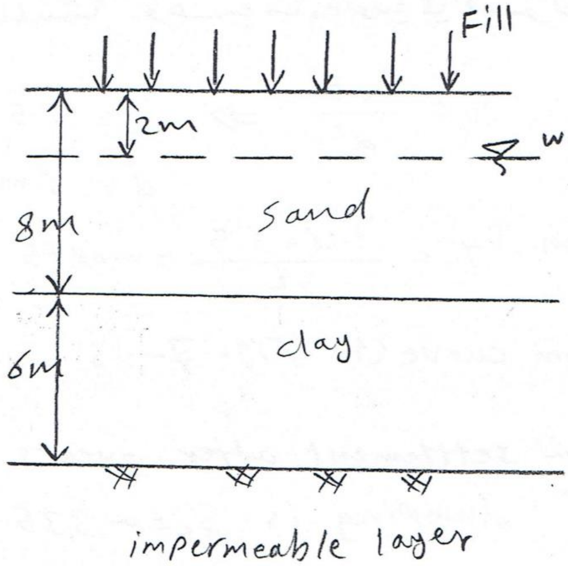
Ex: An 8 m depth of sand overlies a 6m layer of clay, below which is an impermeable stratum (Fig. below). the water table is 2m below the surface of the sand. over a period of 1 year a 3m depth of fill (unit weight 20 kN/m²) is to be dumped on the surface over an extensive area. γ_{sat} of sand 19 kN/m³ and γ_{sat} of clay 20 kN/m³, γ_{sand} above W.T 17 kN/m³. For the clay, the relationship between void ratio and effective stress (units kN/m²) can be represented by the equation $e = 0.88 - 0.32 \log \frac{P'}{100}$ and the coefficient of consolidation is 1.26 m²/year.

a) calculate the final settlement of the area due to consolidation of the clay and the settlement after a period of 3 year from the start of dumping.

b) if a very thin layer of sand, freely draining, existed 1.5 m above the bottom of the clay layer, what would be the values of the final and 3 year settlement?

$C_c = 0.32$





solution

a)

* بيان الدفن يغطي مساحة واسعة ، يمكن اعتبار هذا النوع
من المسائل بأنه one-dimensional problem ، هبوط الانضمام
يمكن حسابه بالاعتماد على قيمة C_c .

$$P_o' = (17*2)+[(19-9.81)*6]+[(20-9.81)*3] = 119.8\ \frac{kN}{m^2}$$

$$e_o = 0.88 - 0.32 \log 1.198 = 0.855$$

$$P_f' = \underbrace{119.8}_{P_o'} + \underbrace{(3*20)}_{\Delta P}\ (h_{fill}\ \gamma_{fill}) = 179.8\ kN/m^2$$

$P_f' \rightarrow P_o',\ \Delta P'$

$$\therefore S_c = \frac{C_c}{1+e_o} \cdot H \log\left(\frac{P_f'}{P_o'}\right)$$

$$= \frac{0.32}{1+0.855} * 6\,m \log \frac{179.8}{119.8} = 0.1825\ m \simeq 182\ mm$$

* لحساب قيمة درجة الانضمام بعد ثلاث سنوات يجب علينا تصحيح
الوقت وكما يلي :-

$$t = 3 - \frac{1}{2} = 2.5\ years$$

ملاحظة/ في مثل هذه الاسئلة يتم تصحيح الوقت وذلك باضافة او طرح
نصف فترة انشاء المنشأ وحسب ماطلوب في السؤال .

$$\therefore T_v = \frac{c_v t}{d^2} \Rightarrow t = 2.5\ years$$

$d = 6\,m \longrightarrow$ half closed

then $T_v = \frac{1.26 * 2.5}{6^2} = 0.0875$

From curve (1) Fig. 7-13 $U = 0.335$

$\therefore$ settlement after 3 years from the start of dumping is $S_c = 0.335 * 182 = 61\ mm$

b) في هذا الفرع نستخدم الانضمام الكلي والذي مقداره 182mm

ولكن الذي يختلف هنا هو معدل حدوث الانضمام سوف يتأثر بأن يكون هناك طبقة نفاذة اسفل الطبقة الطينية وكما موضح في الرسم .

$$\therefore \quad T_v = \frac{c_v\, t}{d^2} = \frac{1.26 * 2.5}{(2.25)^2} = 0.622$$

$$\therefore \quad U_1 = 0.825$$

and $$T_{v_2} = \frac{1.26 * 2.5}{(1.5)^2} = 1.4$$

$$\therefore \quad U_2 = 0.97$$

ملاحظة / في مثل هذه الحالة ولأن طبقة الرمل (sand) السفلية مذكوراً في السؤال بأنها طبقة thin فان الحل يتطلب ايجاد U لطبقة الطين بالابعاد الجديدة وكذلك U لطبقة ال sand ال thin ثم نوجد ال U المكافئة وكما يلي :-

$$\bar{U} = 4.5 * U_1 + 1.5\, U_2$$

$$= 4.5 * 0.825 + 1.5 * 0.97 = 5.1675$$

$$\therefore U = \frac{5.1675}{6} = 0.86$$

$\therefore$ the 3 year settlement is

$$S_c = 0.86 * 182 = \underline{\underline{157\ \text{mm}}}$$

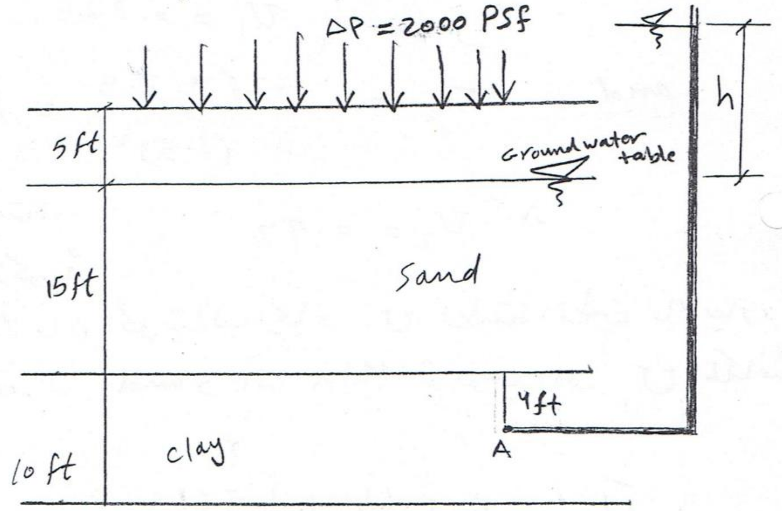
Ex A soil profile is shown in Fig. below. A surcharge load of 2000 Ib/ft^2 is applied on the ground surface.

Determine the following.

a- How high the water will rise in the piezometer immediatly after the application of load.

b- what the degree of consolidation is at A when h = 20 ft.

c- find h when the degree of consolidation at A is 50 %



Solution:-

Part (a) ; Assuming uniform increas of initial excess pore water pressure through th 10 ft depth of clay layer.

$$u_o = \Delta P = 2000\ Ib/ft^2$$

$$h = \frac{2000}{62.4} = \underline{32.05}\ ft$$

Part (b) $\quad U_A\% = \left(1 - \frac{U_A}{u_o}\right)100 = \left(1 - \frac{20 * 62.4}{32.05 * 62.4}\right) = \underline{37.6\%}$

Part (C) $\quad U_A = 0.5 = \left(1 - \frac{U_A}{u_o}\right)$

or $\quad 0.5 = \left(1 - \frac{U_A}{2000}\right) \quad \therefore U_A = (1 - 0.5)2000 = 1000\ Ib/ft^2$

Hence $\quad h = \frac{1000}{62.4} = \underline{16.03\ ft}$

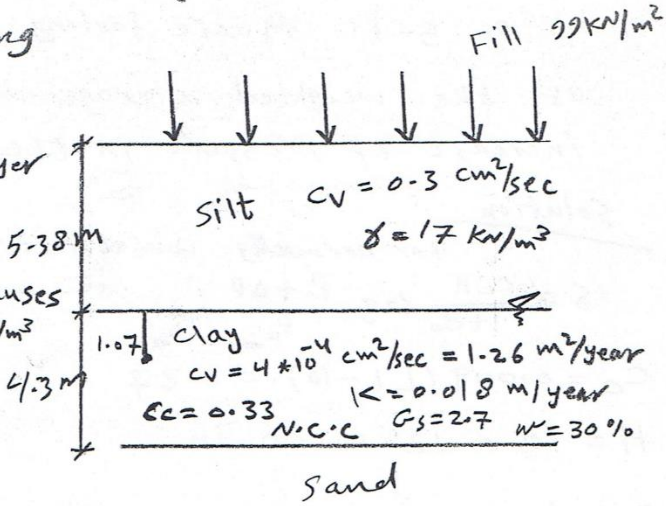
Ex: The stratum of clay and loading shown in Fig. below :-

a) at $z = 1.07$ m from the top of clay layer and 4 months after loading find excess pore pressure.

b) total settlement of clay layer

c) time of 73 % consolidation

d) what is the hight of Fill causes 15cm settlement if $\gamma_{Fill} = 18\ kN/m^3$



Solution :

because of the overlying and underlying soils are much more permeable than the clay, there is double drainage (open layer).

$$\therefore H_{dr} = \frac{4.3}{2} = 2.1 \ m$$

$$\therefore \frac{z}{H_{dr}} = \frac{1.07}{2.1} = 0.5$$

$$T_v = \frac{c_v t}{H_{dr}^2} = \frac{1.26\ m^2/year * \frac{4}{12}\ year}{(2.1)^2} = 0.09$$

From Fig. 7.24 $\frac{z}{H_{dr}} = 0.5$, $T_v = 0.09$

$$\therefore U_z = 0.24$$

$$U_z = 1 - \frac{u_z}{u_o} = 0.24$$

$$\therefore u_z = u_o (1 - 0.24)$$

$$u_o = \Delta P = 99\ KN/m^2$$

$$\therefore u_z = 99 (1 - 0.24) = 75.2\ KN/m^2 \rightarrow (a)$$

$$S = \frac{C_c H}{1+e_o} \log \frac{P_f}{P_o}$$

$$Gw = S \cdot e$$

$$e = \frac{2.7 * 0.3}{1} = 0.81$$

$$\gamma_{sat\ clay} = \frac{G_s + S \cdot e}{1+e} \gamma_w = \frac{2.7 + 1 * 0.81}{1 + 0.81} * 9.81 = 19.024 \ kN/m^3$$

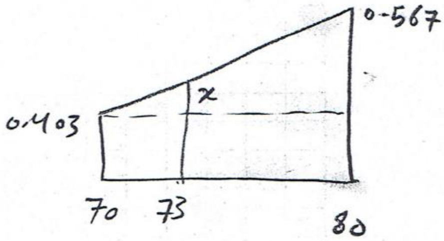
$$P_o = 5.38 * 17 + \frac{4.3}{2} (19.024 - 9.81) = 111.27 \ kN/m^2$$

$$\Delta P = 99 \ kN/m^2$$

$$\therefore S = \frac{0.33 * 4.3}{1 + 0.81} \log \frac{111.27 + 99}{111.27} = \underline{\underline{21.67 \ cm}} \rightarrow (b)$$

u_o constant with depth (Fill).
For $U = 73\%$ From table 7-3 or Fig. 7.13 curve (1)

$$T_v = 0.4522$$



$$\therefore T_v = \frac{C_v t}{H_{dr}^2}$$

$$\frac{x}{3} = \frac{0.567 - 0.403}{10}$$

$$x = 0.0492$$

$$\therefore T_v = 0.403 + 0.0492 = 0.4522$$

$$\therefore t = \frac{0.4522 * \left(\frac{4.3}{2}\right)^2 \cancel{m^2}}{1.26 \ \cancel{m^2}/year}$$

$$= \underline{\underline{1.658}} \ year \rightarrow (c)$$

$$S = \frac{0.33 * 4.3}{1 + 0.81} \log \frac{P_f}{111.27} = 0.15$$

$$\therefore \log \frac{P_f}{111.27} = 0.19133$$

$$\therefore \frac{P_f}{111.27} = 1.5535$$

$$\therefore P_f = 172.866 \ kN/m^2$$

$$\because P_f = \Delta P + P_o \quad \therefore \Delta P = 172.866 - 111.27 = 61.59 \ kN/m^2$$

$$\because \Delta P = Fill = \gamma_{fill} * h_{fill} \quad \therefore 61.59 = 18 * h$$

$$\therefore h = \frac{61.59 \ kN/m^2}{18 \ kN/m^3} = \underline{3.422} \ m \rightarrow (d)$$